

Effect of Radiation and Dissipation on Unsteady Convective Heat Transfer flow of a Viscous electrically conducting fluid in a Vertical Channel with Non-linear density temperature variation

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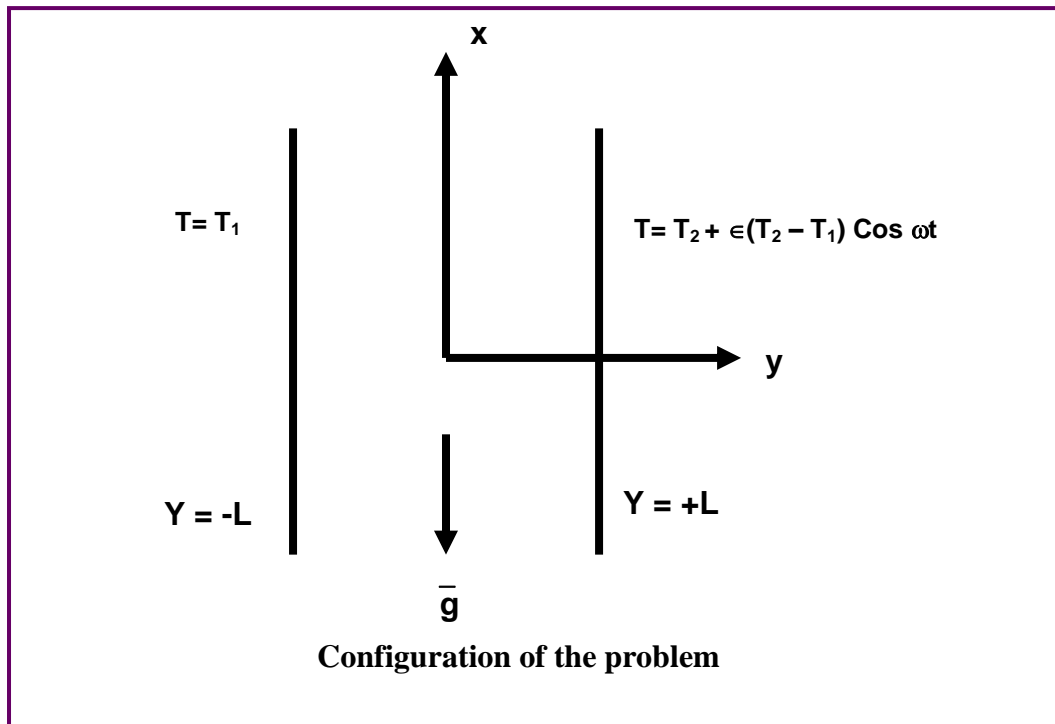
ABSTRACT: We analyse the unsteady convective Heat Transfer flow of a viscous fluid in a Vertical channel on whose walls an oscillatory temperature is prescribed. Approximate solutions to coupled non-linear partial differential equations governing the flow, heat transfer are solved by a perturbation technique. The velocity, temperature, skin friction, and rate of heat transfer are discussed for different variations of G , M , α , N , γ , γ_1 , P and Ec .

Key words : Heat transfer, Dissipation, Radiation.

1. Introduction

The process of free convection as a mode of heat transfer has wide applications in the fields of Chemical Engineering, Aeronautical and Nuclear power generation. It was shown by Gill and Casal (6) that the buoyancy significantly affects the flow of low Prandtl number fluids which is highly sensitive to gravitational force and the extent to which the buoyancy force influences a forced flow is a topic of interest. Free convection flows between two long vertical plates have been studied for many years because of their engineering applications in the fields of nuclear reactors, heat exchangers, cooling appliances in electronic instruments. These flows were studied by assuming the plates at two different constant temperatures or temperature of the plates varying linearly along the plates etc. The study of fully developed free convection flow between two parallel plates at constant temperature was initiated by Ostrach (16). Combined natural and forced convection laminar flow with linear wall temperature profile was also studied by Ostrach (17). The first exact solution for free convection in a vertical parallel plate channel with asymmetric heating for a fluid of constant properties was presented by Anug (1). Many of the early works on free convection flows in open channels have been reviewed by Manca *et al.* (11). Recently, Campo *et al.* (2) considered natural convection for heated iso-flux boundaries of the channel containing a low-Prandtl number fluid. Pantokratoras (18) studied the fully developed free convection flow between two asymmetrically heated vertical parallel plates for a fluid of varying thermophysical properties. However, all the above studies are restricted to fully developed steady state flows. Very few papers deal with unsteady flow situations in vertical parallel plate channels. Transient free convection flow between two long vertical parallel plates maintained at constant but unequal temperatures was studied by Singh *et al.* (20). Jha *et al.* (9) extended the problem to consider symmetric heating of the channel walls. Narahari *et al.* (15) analyzed the transient free convection flow between two long vertical parallel plates with constant heat flux at one boundary, the other being maintained at a constant temperature. Singh and Paul (20) presented an analysis of the transient free convective flow of a viscous incompressible fluid between two parallel vertical walls occurring as a result of asymmetric heating / cooling of the walls. Narahari (14) presented an exact solution to the problem of unsteady free convective flow of a viscous incompressible fluid between two long vertical parallel plates with the plate temperature linearly varying with time at one boundary, the other boundary being held at constant. There are many reasons for the flow to become unsteady. When the current is periodic due to on-off control mechanisms or due to partially rectified *a-c* voltage, there exist periodic heat inputs. Hence, it is important to study the effects of periodic heat flux on the unsteady natural convection, imposed on one of the plates of a channel formed by two long vertical parallel plates, the other being held at a constant initial fluid temperature. Recently Narahari (15) has discussed the unsteady free convection flow of dissipative viscous incompressible fluid between two long vertical parallel plates in which the temperature of one of the plates is oscillatory whereas that of the other plate is uniform. Recently Haritha (7) has analysed unsteady convective heat transfer of dissipative viscous fluid through a porous medium confined in a vertical channel on whose walls an oscillatory temperature is prescribed. Recently Ibrahim *et al.* (8) have studied the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable

moving plate with heat source and suction Recently Kesavaiah et al (10) have studied the effect of the chemical reaction and radiation absorption on an unsteady MHD convective Heat and Mass Transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction.



2. Formulataion of the problem

We consider the flow of a viscous incompressible chemically reacting fluid in a vertical channel bounded by flat walls in the presence of constant heat sources. We choose a Cartesian coordinate system $O(x, y)$ with walls at $y = \pm 1$ by using Boussinesq approximation we consider the density variation only on the buoyancy term. The equation governing the flow to heat and mass transfer are

Equation of Linear Momentum

$$\frac{\partial u}{\partial t} = \frac{\mu}{\rho_0} \frac{\partial^2 u}{\partial y^2} - \rho \bar{g} - (\sigma \mu_e^2 H_0^2) u \quad 2.1$$

Equation of Energy

$$\rho_0 c_p \left[\frac{\partial T}{\partial t} \right] = k_f \frac{\partial^2 T}{\partial y^2} + Q + \mu u_y^2 - \frac{\partial(q_R)}{\partial y} \quad 2.2$$

Equation of State

$$\rho - \rho_0 = -\beta_0(T - T_0) - \beta_1(T - T_0)^2 \quad 2.3$$

where u is the velocity component in x -direction, T is the temperature, p is the pressure, ρ is the density, σ is the electrically conductivity, μ_e is the magnetic permeability, k is the coefficient of porous permeability, μ is dynamic viscosity, k_f is coefficient of thermal conductivity β is the coefficient of volume expansion, Q is the strength of heat source,

The boundary conditions are

$$\begin{aligned} u = 0, T = T_1, \text{ at } y = -L \\ u = 0, T = T_1 + \epsilon(T_2 - T_1) \cos(\omega t), \text{ at } y = +L \end{aligned} \quad (2.4)$$

On introducing the non-dimensional variables.

$$u' = \frac{u}{v/L}, \quad y' = y/L, \quad \theta = \frac{T - T_1}{T_2 - T_1}, \quad t' = \omega t,$$

Equations (2.1) – (2.2) reduce to (dropping the dashes)

$$\gamma^2 \frac{\partial u}{\partial t} = G[\theta + \gamma_1 \theta^2] + \frac{\partial^2 u}{\partial y^2} - M^2 u \quad (2.5)$$

$$P\gamma^2 \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + \alpha + PE_c u_y^2 + \frac{4}{3N_1} \frac{\partial^2 \theta}{\partial y^2} \quad (2.6)$$

where

$$G = \beta g L^3 \frac{(T_2 - T_1)}{v^2} \quad (\text{Grashoff number})$$

$$P = \frac{\mu C_p}{K_f} \quad (\text{Prandtl number})$$

$$\alpha = \frac{QL^2}{(T_2 - T_1)K_f} \quad (\text{Heat source parameter})$$

$$N_1 = \frac{3\beta_R}{4\sigma^* T_e^3} \quad (\text{Radiation parameter})$$

$$E_c = \frac{\beta g L^3}{C_p} \quad (\text{Eckert number})$$

$$\gamma = \frac{\omega L^2}{\nu} \quad (\text{Wormsly number})$$

The transformed boundary conditions are

$$\begin{aligned} u = 0, \quad \theta = 0, \quad \text{at } y = -1 \\ u = 0, \quad \theta = 1 + \varepsilon \cos(\omega t), \quad \text{at } y = +1 \end{aligned} \quad (2.7)$$

3. Shear stress and Nusselt number

The shear stress at the wall $y = \pm 1$ is given by

$$\tau = \left(\frac{\partial u}{\partial y} \right)_{y=\pm 1}$$

and the corresponding equations are

$$(\tau)_{y=-1} = a_{14} + a_{15} + Ec[a_{16} + a_{17} + a_{18} + a_{19}] + (0.01).E_{33}.a_{20}$$

$$(\tau)_{y=+1} = (a_7 + a_8) + Ec[a_9 + a_{10} + a_{11} + a_{12}] + (0.01).E_{33}.a_{13}$$

The rate of heat transfer (Nusselt number) at the walls $y = \pm 1$ is given by

$$(Nu)_{y=\pm 1} = \left(\frac{d\theta}{dy} \right)_{y=\pm 1}$$

and the corresponding expressions are

$$(Nu)_{y=-1} = a_{27} + Ec[a_{28} + a_{29} + a_{30}Sh\beta_1 + a_{31}Ch\beta_1] + (0.01).E_{33}.a_{32}$$

$$(Nu)_{y=+1} = a_{21} + Ec[a_{22} + a_{23} + a_{24}Sh\beta_1 + a_{25}Ch\beta_1] + (0.01).E_{33}.a_{26}$$

where a_1, a_2, \dots, a_{32} are constants

4. Discussion of the numerical results

In this analysis we investigate the effect of radiation, dissipation and non – linear density temperature variation on convective heat transfer flow of a viscous , electrically conducting fluid in a vertical channel in the presence of heat sources .

Figs.1-8 represent the axial velocity u with different values of G , M , α , N , γ , γ_1 , P and Ec . Fig.1 represents u with Grashof number G . It is found that u is in the vertically downwards for $G>0$ and vertically upwards for $G<0$ with maximum attained at $y = -0.6$. $|u|$ enhances with increase in $|G|$. The variation of u with magnetic parameter M shows that higher the Lorentz forces larger $|u|$ in the flow region.(fig.2). Fig.3 represents u with heat source parameter α . It is found that $u<0$ for $\alpha >0$ and $u > 0$ for $\alpha < 0$. $|u|$ experiences and enhancement with increase in the strength of the heat source / sink with respect to radiation parameter N we find that higher the radiative heat flux larger $|u|$ in the flow region (fig.4). An increase in the Wormsely number $\gamma \leq 4$ reduces $|u|$ and enhances with higher $\gamma \geq 6$ (fig.5). The variation of u with density ratio γ_1 shows that $|u|$ enhances with γ_1 in the right half and depreciates in the left half of the channel (fig.6). with respect to the Prandtl number P we find that $|u|$ depreciates with $P \leq 1$ and enhances with higher values of $P \geq 7$ (fig.7). From fig.8 we find that higher the dissipative heat larger $|u|$ everywhere in the flow region except in a narrow region adjacent to $y = +1$ where it depreciates.

The non – dimensional temperature (θ) is exhibited in figs. 9 – 16 for different parametric values. Fig.9 represents the temperature θ with Grashof number G . We follow the convention that the non – dimensional temperature is positive or negative according as the actual temperature is greater / lesser than T_1 . It is found that the actual temperature enhances with increase in $|G| (>_< 0)$ everywhere in the flow region . From fig.10 notice that higher the Lorentz force larger the actual temperature and for further higher Lorentz forces smaller the actual temperature. With respect to the heat source parameter α we find that the actual temperature enhances with the strength of the heat source ($\alpha >0$) and reduces with that of heat sink ($\alpha <0$) (fig.11). Higher the radiative heat flux larger the actual temperature in the entire flow region (fig.12). An increase in the Wormsely number γ results in a depreciation in θ (fig.13). The effect of non – linear density variation on θ is shown in fig.14. It is found that the actual temperature depreciates with increase in the density ratio γ_1 . The variation of θ with Prandtl number P shows that the actual temperature depreciates with $P \leq 1$ and enhances with $P \geq 7$ (fig.15). From fig.16 we find that higher the dissipative heat larger the actual temperature everywhere in the flow region.

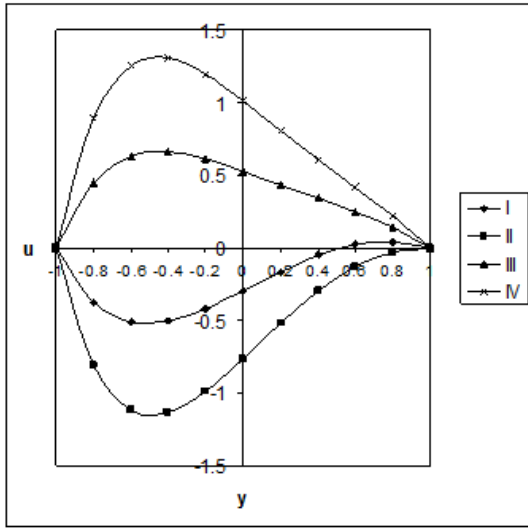


Fig. 1: Variation of u with G

$M=2, \alpha=2, N=0.5, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV |
|-----|--------|-----------------|---------|------------------|
| G | 10^3 | 2×10^3 | -10^3 | -2×10^3 |

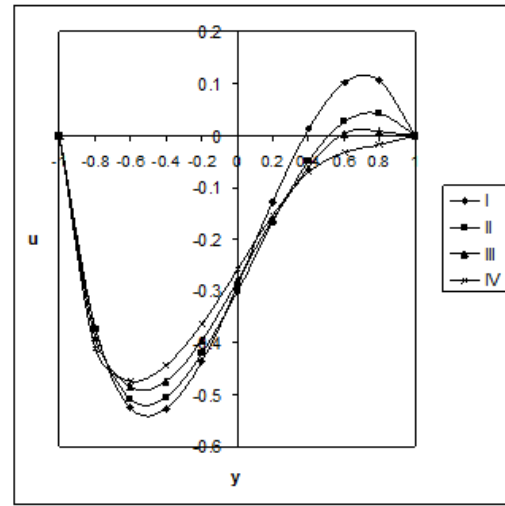


Fig. 2: Variation of u with M

$G=10^3, \alpha=2, N=0.5, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV |
|-----|---|----|-----|----|
| M | 2 | 4 | 6 | 10 |

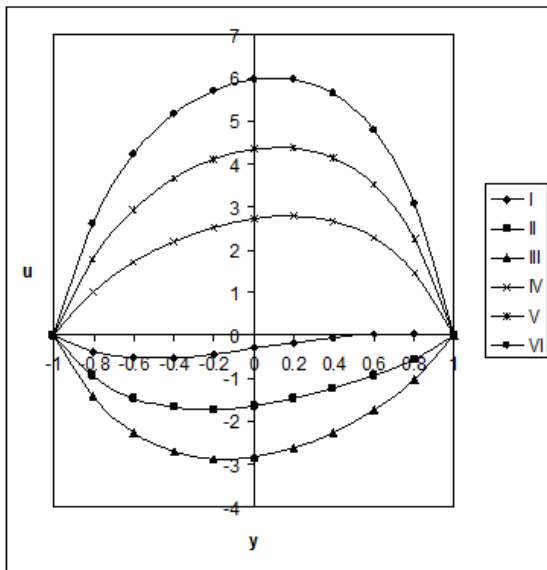


Fig. 3: Variation of u with α

$G=10^3, M=2, N=0.5, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV | V | VI |
|----------|---|----|-----|----|----|----|
| α | 2 | 4 | 6 | -2 | -4 | -6 |

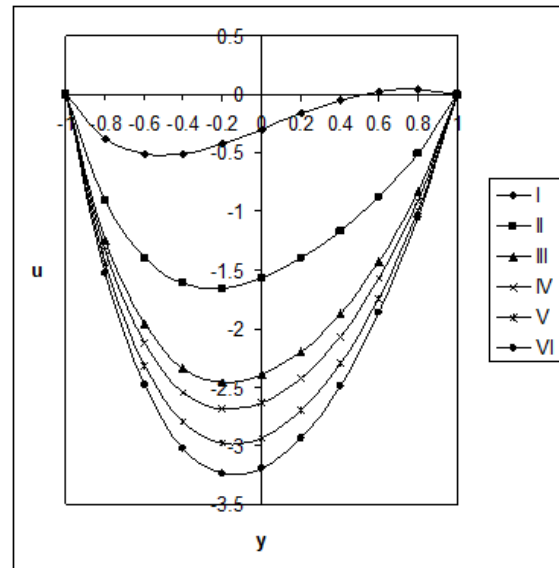


Fig. 4: Variation of u with N

$G=10^3, M=2, \alpha=2, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV | V | VI |
|-----|-----|-----|-----|----|----|-----|
| N | 0.5 | 1.5 | 2.5 | 5 | 10 | 100 |

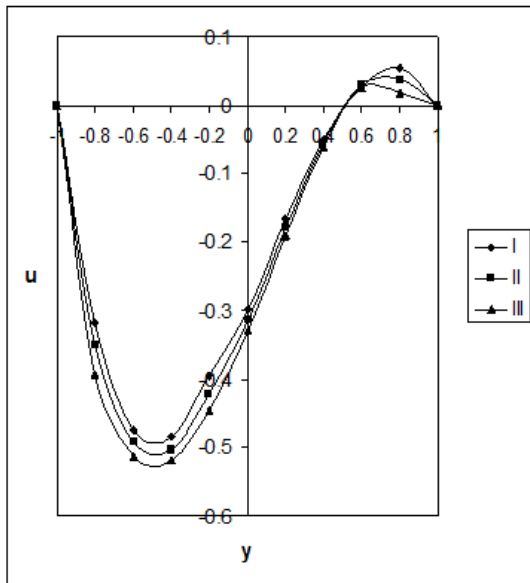


Fig. 5.: Variation of u with γ
 $G=10^3, M=2, \alpha=2, N=0.5, \gamma_1=2, Ec=0.004$

| | I | II | III |
|----------|---|----|-----|
| γ | 2 | 4 | 6 |

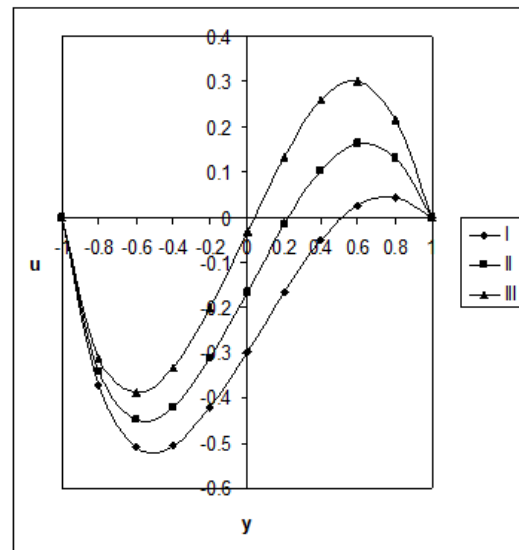


Fig. 6.: Variation of u with γ_1
 $G = 10^3, M=2, \alpha=2, N=0.5, \gamma=2, Ec=0.004$

| | I | II | III |
|------------|---|----|-----|
| γ_1 | 2 | 4 | 6 |

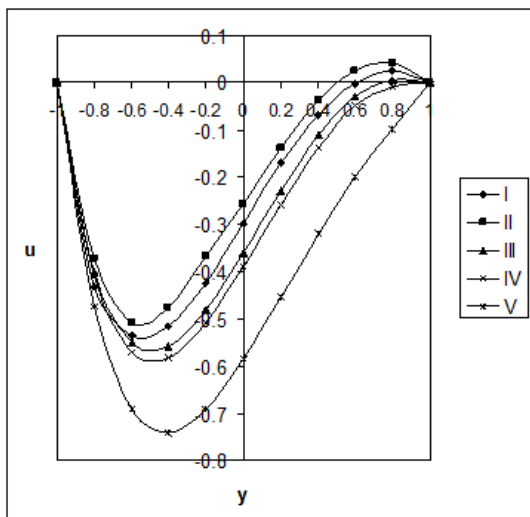


Fig. 7.: Variation of u with P
 $G=10^3, M=2, \alpha=2, N=0.5, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV | V |
|-----|------|------|-----|----|----|
| P | 0.01 | 0.71 | 7 | 10 | 30 |

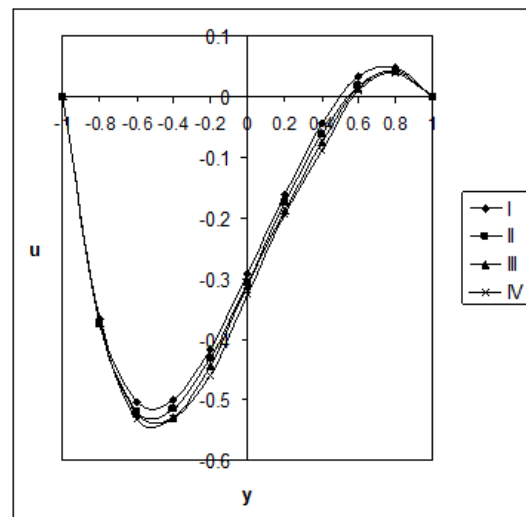


Fig. 8.: Variation of u with Ec
 $G = 10^3, M=2, \alpha=2, N=0.5, \gamma=2, \gamma_1=2, P=0.71$

| | I | II | III | IV |
|------|-------|-------|-------|-------|
| Ec | 0.001 | 0.004 | 0.006 | 0.009 |

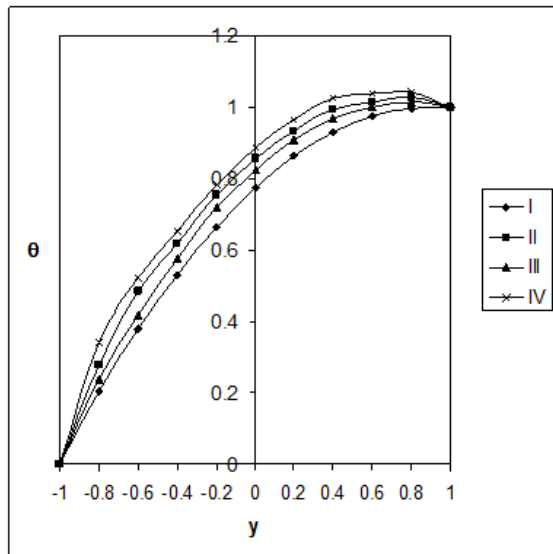


Fig. 9 : Variation of θ with G
 $M=2, \alpha=2, N=0.5, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV |
|-----|--------|-----------------|--------|-----------------|
| G | 10^3 | 2×10^3 | 10^3 | 2×10^3 |

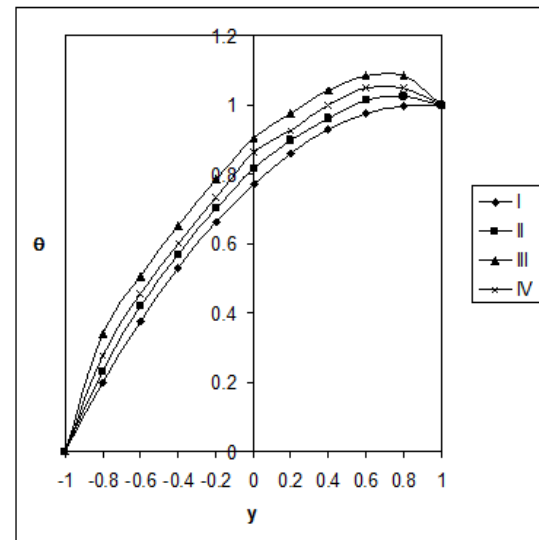


Fig. 10 : Variation of θ with M
 $\alpha=2, N=0.5, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV |
|-----|---|----|-----|----|
| M | 2 | 4 | 6 | 10 |

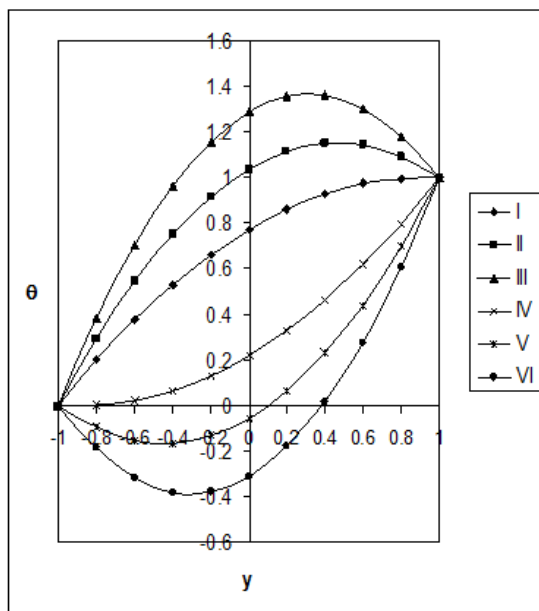


Fig. 11 : Variation of θ with α
 $G=10^3, M=2, N=0.5, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV | V | VI |
|----------|---|----|-----|----|----|----|
| α | 2 | 4 | 6 | -2 | -4 | -6 |

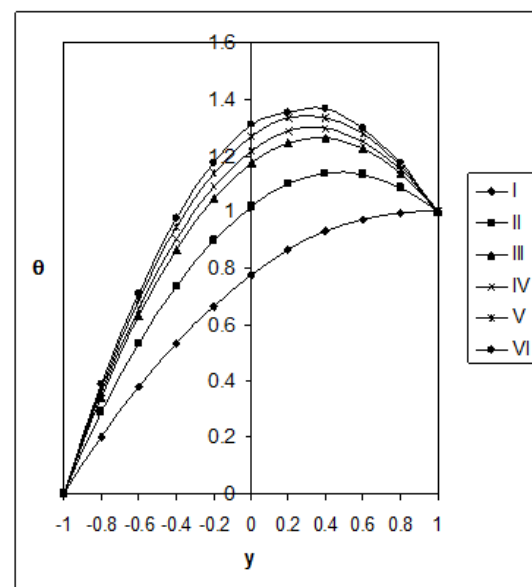


Fig. 12 : Variation of θ with N
 $G=10^3, M=2, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV | V | VI |
|-----|-----|-----|-----|----|----|-----|
| N | 0.5 | 1.5 | 2.5 | 5 | 10 | 100 |

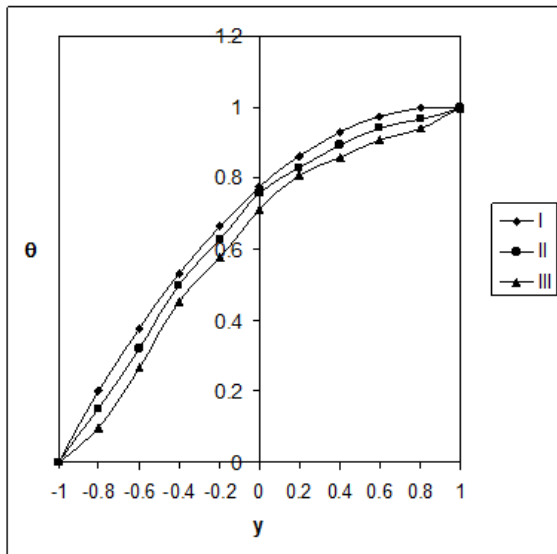


Fig. 13: Variation of θ with γ
 $G=10^3, M=2, \alpha=2, N=0.5, \gamma_1=2, Ec=0.004$

| | I | II | III |
|----------|---|----|-----|
| γ | 2 | 4 | 6 |

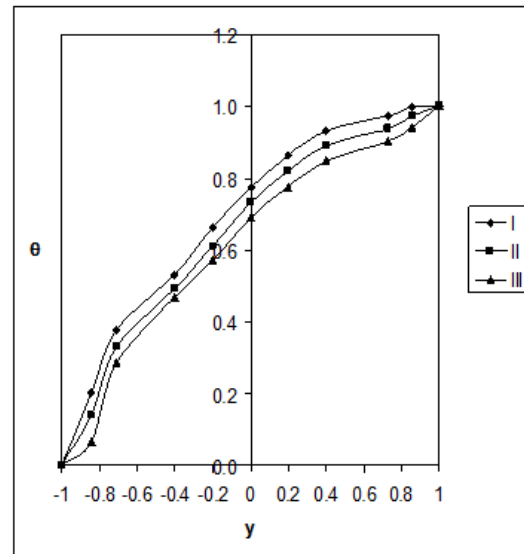


Fig. 14: Variation of θ with γ_1
 $G=10^3, M=2, \alpha=2, N=0.5, \gamma=2, Ec=0.004$

| | I | II | III |
|------------|---|----|-----|
| γ_1 | 2 | 4 | 6 |

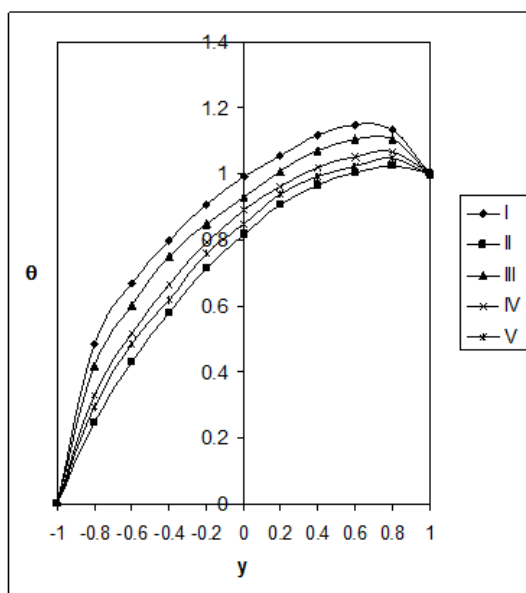


Fig. 15: Variation of θ with P
 $G=10^3, M=2, \alpha=2, N=0.5, \gamma=2, \gamma_1=2, Ec=0.004$

| | I | II | III | IV | V |
|-----|------|------|-----|----|----|
| P | 0.01 | 0.71 | 7 | 10 | 30 |

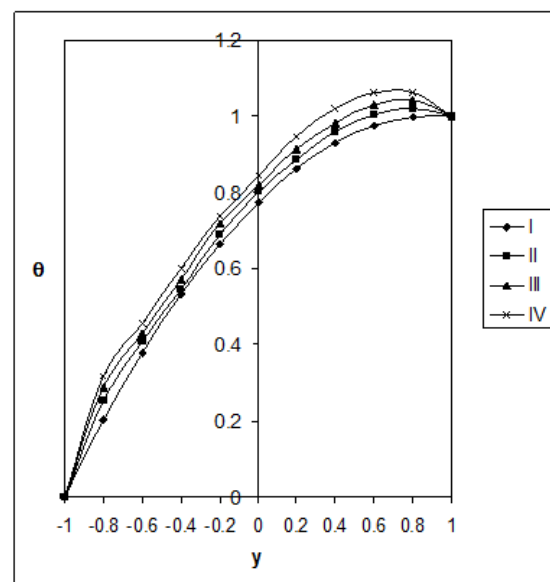


Fig. 16: Variation of θ with Ec
 $G=10^3, M=2, \alpha=2, N=0.5, \gamma=2, \gamma_1=2, P=0.71$

| | I | II | III | IV |
|------|-------|-------|-------|-------|
| Ec | 0.001 | 0.004 | 0.006 | 0.009 |

The shear stress (τ) at $y = \pm 1$ is exhibited in tables 1- 6 for different values of $G, M, \alpha, N, \gamma, \gamma_1, P$ and Ec . It is found that the stress enhances with increase in $|G|$ at both the walls. $|\tau|$ depreciates with increase in M $y = +1$ and at $y = -1, |\tau|$ reduces with $M \leq 4$ and enhances with higher $M \geq 6$ in the heating case and in the cooling case it reduces with M . An increase in the strength of the heat source/sink results in an enhancement in $|\tau|$ at $y = +1$ and depreciation at $y = -1$ (tables 1&4). The variation of the stress with radiation parameter N shows that higher the radiative heat flux larger $|\tau|$ at both the walls. An increase in the Wormsely number γ reduces $|\tau|$ for $G > 0$ and enhances it for $G < 0$ at $y = \pm 1$. With respect to density ratio γ_1 we find that $y = -1$ in the heating case and in the cooling case, it enhances at $y = +1$ and reduces at $y = -1$ (tables 2&5). The variation of τ with Prandtl number P shows that $|\tau|$ enhances with increase in P at $y = +1$ and at $y = -1$, it enhances in the heating case and reduces in the cooling case. Also higher the dissipative heat larger $|\tau|$ at $y = +1$ and depreciates at $y = -1$. The rate of heat transfer (Nusselt number) at $y = \pm 1$ is shown in tables 7-12 for different parametric values. It is found that the rate of heat transfer enhances with increase in $|G|$ at both the walls. Higher the Lorentz force lesser the rate of heat transfer at $y = \pm 1$. The variation of Nu with heat source parameter α shows that $|Nu|$ reduces with $\alpha \leq 4$ and enhances with higher $\alpha \geq 6$ while it enhances at $y = \pm 1$ with the strength of the heat sink (tables 7&10). With respect to radiation parameter N we find that higher the radiative heat flux larger the rate of heat transfer at both the walls. The variation of Nu with Wormsely number γ shows that $|Nu|$ reduces with γ in the heating case and in the cooling case, it enhances with γ at $y = +1$. At $y = -1$ $|Nu|$ enhances with γ for all G . $|Nu|$ reduces with the density ratio γ for $G > 0$ and enhances for $G < 0$ at $y = \pm 1$ (tables 8&11). From tables 9 & 12 we find that the rate of heat transfer enhances with increase in P and Ec . Thus higher the dissipative heat larger $|Nu|$ at both the wall.

TABLE – 1 SHEAR STRESS (τ) AT $y = +1$

| G | I | II | III | IV | V | VI | VII | VIII |
|----------------------------|-------------------|------------------|-----------------|------------------|------------------|------------------|------------------|------------------|
| 1x10³ | 105.73640 | 21.86104 | 4.30267 | 54.32152 | 101.60240 | -78.36313 | -97.45152 | -92.15113 |
| 2x10³ | 982.83280 | 216.16750 | 32.72691 | 220.83170 | 434.95960 | - | -116.38720 | 61.71305 |
| - | -144.15270 | -31.91592 | -6.63404 | -66.98370 | - | 78.08633 | 114.83790 | 155.61720 |
| 1x10³ | | | | | 148.07680 | | | |
| - | - | - | - | - | - | 172.61130 | 205.83210 | 223.82820 |
| 2x10³ | 1135.04500 | 252.43270 | 37.87989 | 265.82350 | 610.34820 | | | |
| M | 2 | 4 | 6 | 6 | 6 | 6 | 6 | 6 |
| α | 2 | 2 | 2 | 4 | 6 | -2 | -4 | -6 |

TABLE – 2 SHEAR STRESS (τ) AT $y = +1$

| G | I | II | III | IV | V | VI | VII | VIII | IX |
|------------------|------------|------------|------------|-----------|-----------|------------|------------|-------------|------------|
| 1×10^3 | 4.30267 | 64.79616 | 121.61490 | 141.21860 | 168.06270 | 4.05908 | 3.82962 | 4.99012 | 4.67103 |
| 2×10^3 | 32.72691 | 74.51570 | 144.46730 | 164.58220 | 208.67700 | 31.5211 | 31.19681 | 34.09786 | 33.46155 |
| -1×10^3 | -6.63404 | -85.46013 | - | - | - | -6.90215 | -7.16687 | -5.96448 | -6.27873 |
| -2×10^3 | -37.87989 | -144.70100 | 100.19410 | 125.90340 | 150.67650 | -38.70372 | -39.52956 | -36.54471 | -37.17134 |
| N | 0.5 | 1.5 | 3.5 | 5 | 10 | 0.5 | 0.5 | 0.5 | 0.5 |
| γ | 2 | 2 | 2 | 2 | 2 | 4 | 6 | 2 | 2 |
| γ_1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0.5 | 1.2 |

TABLE – 3 SHEAR STRESS (τ) AT $y = +1$

| G | I | II | III | IV | V | VI |
|------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1×10^3 | 0.78191 | 4.30267 | 36.04801 | 51.19926 | 6.09516 | 8.78389 |
| 2×10^3 | 2.57575 | 32.72691 | 303.76550 | 433.04700 | 48.02482 | 70.97167 |
| -1×10^3 | -2.15911 | -6.63404 | -46.73572 | -65.85173 | -8.89651 | -12.29022 |
| -2×10^3 | -3.99269 | -37.87989 | -342.27190 | -487.44040 | -55.05858 | -80.82662 |
| P | 0.01 | 0.71 | 7 | 10 | 0.71 | 0.71 |
| Ec | 0.004 | 0.004 | 0.004 | 0.004 | 0.006 | 0.009 |

TABLE – 4 SHEAR STRESS (τ) AT $y = -1$

| G | I | II | III | IV | V | VI | VII | VIII |
|-----------------|------------|-----------|------------|-----------|-----------|-----------|------------|-------------|
| 1×10^3 | -58.68925 | -22.47087 | - | -71.64146 | - | 11.33958 | -17.73453 | -91.85069 |
| 2×10^3 | -473.16980 | -54.64562 | 25.02542 | - | 125.63680 | - | - | - |
| - | 125.51620 | 38.47137 | 36.11290 | 208.23520 | 501.00330 | 196.54790 | 603.38640 | 1279.4500 |
| 1×10^3 | - | - | 27.46671 | 90.42934 | 193.38780 | -17.85514 | -17.96113 | -13.98139 |
| 2×10^3 | 742.37660 | 118.17720 | 44.22469 | 279.73290 | 762.15420 | 163.49560 | 445.94620 | 830.63600 |
| M | 2 | 4 | 6 | 6 | 6 | 6 | 6 | 6 |
| α | 2 | 2 | 2 | 4 | 6 | -2 | -4 | -6 |

TABLE – 5 SHEAR STRESS (τ) AT $y = -1$

| G | I | II | III | IV | V | VI | VII | VIII | IX |
|-----------------|------------|------------|------------|------------|------------|------------|------------|-------------|------------|
| 1×10^3 | - | -72.67495 | -131.83450 | -156.38110 | -194.74970 | - | - | - | - |
| 2×10^3 | 25.02542 | - | -726.78500 | -963.64600 | - | 23.77811 | 22.53237 | 24.98815 | 25.00319 |
| - | - | - | 36.11290 | 255.79040 | 1377.13400 | 31.30471 | 26.50625 | 35.53059 | 35.79753 |
| 1×10^3 | 27.46671 | 104.87210 | 250.12320 | 325.90080 | 464.11790 | 28.73800 | 30.02402 | 27.47224 | 27.46711 |
| 2×10^3 | 44.22469 | 380.75230 | 1192.43500 | 1632.58100 | 2442.76800 | 49.08082 | 53.95985 | 44.74349 | 44.49638 |
| N | 0.5 | 1.5 | 3.5 | 5 | 10 | 0.5 | 0.5 | 0.5 | 0.5 |
| γ | 2 | 2 | 2 | 2 | 2 | 4 | 6 | 2 | 2 |
| γ_1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0.5 | 1.2 |

TABLE – 6 SHEAR STRESS (τ) AT $y = -1$

| G | I | II | III | IV | V | VI |
|------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1×10^3 | -27.96713 | -25.02542 | 1.36673 | 13.95117 | -23.53621 | -21.30240 |
| 2×10^3 | -56.00413 | -36.11290 | 142.58280 | 227.80810 | -26.02787 | -10.90032 |
| -1×10^3 | 28.53388 | 27.46671 | 17.83624 | 13.23979 | 26.92285 | 26.10705 |
| -2×10^3 | 56.65075 | 44.22469 | -67.47360 | -120.75110 | 37.92023 | 28.46356 |
| P | 0.01 | 0.71 | 7 | 10 | 0.71 | 0.71 |
| Ec | 0.004 | 0.004 | 0.004 | 0.004 | 0.006 | 0.009 |

TABLE – 7 NUSSELT NUMBER (Nu) AT $y = +1$

| G | I | II | III | IV | V | VI | VII | VIII |
|------------------|-----------|-----------|------------|-----------|-----------|-----------|------------|-------------|
| 1×10^3 | -6.32839 | -3.12718 | -1.14483 | -2.92962 | -4.43624 | 2.45303 | 3.84583 | 4.67620 |
| 2×10^3 | -26.62394 | -12.87747 | -4.54380 | -10.62925 | -16.50824 | 6.60260 | 10.82440 | 13.57437 |
| -1×10^3 | -7.84515 | -3.66128 | -1.26145 | -3.64372 | -6.69464 | 2.36642 | 4.02942 | 5.85861 |
| -2×10^3 | -29.65745 | -13.94568 | -4.77702 | -12.05746 | -21.02504 | 6.42936 | 11.19158 | 15.93919 |
| M | 2 | 4 | 6 | 6 | 6 | 6 | 6 | 6 |
| α | 2 | 2 | 2 | 4 | 6 | -2 | -4 | -6 |

TABLE – 8 NUSSELT NUMBER (Nu) AT $y = +1$

| G | I | II | III | IV | V | VI | VII | VIII | IX |
|------------------|------------|------------|------------|-----------|-----------|------------|------------|-------------|------------|
| 1×10^3 | - | -4.96402 | -8.93808 | - | - | - | - | - | - |
| | 1.14483 | | | 10.38428 | 12.46329 | 1.11993 | 1.09511 | 1.16942 | 1.15793 |
| 2×10^3 | - | - | - | - | - | - | - | - | - |
| | 4.54380 | 19.41147 | 37.00123 | 44.00551 | 54.80855 | 4.49394 | 4.44417 | 4.59340 | 4.57023 |
| -1×10^3 | - | -6.23907 | - | - | - | - | - | - | - |
| | 1.26145 | | 13.16732 | 16.28607 | 21.53738 | 1.28644 | 1.31153 | 1.23600 | 1.24785 |
| -2×10^3 | - | - | - | - | - | - | - | - | - |
| | 4.77702 | 21.96156 | 45.45970 | 55.80908 | 72.95672 | 4.82697 | 4.87700 | 4.72656 | 4.75009 |
| N | 0.5 | 1.5 | 3.5 | 5 | 10 | 0.5 | 0.5 | 0.5 | 0.5 |
| γ | 2 | 2 | 2 | 2 | 2 | 4 | 6 | 2 | 2 |
| γ_1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0.5 | 1.2 |

TABLE – 9 NUSSELT NUMBER (Nu) AT $y = +1$

| G | I | II | III | IV | V | VI |
|------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1×10^3 | -0.06154 | -1.14483 | -10.84045 | -15.46805 | -1.69221 | -2.51328 |
| 2×10^3 | -0.10941 | -4.54380 | -45.35136 | -63.34079 | -6.79066 | -10.16095 |
| -1×10^3 | -0.06318 | -1.26145 | -11.99015 | -17.11048 | -1.86713 | -2.77566 |
| -2×10^3 | -0.11270 | -4.77702 | -46.65076 | -66.62566 | -7.14050 | -10.68571 |
| P | 0.01 | 0.71 | 7 | 10 | 0.71 | 0.71 |
| Ec | 0.004 | 0.004 | 0.004 | 0.004 | 0.006 | 0.009 |

TABLE – 10 NUSSELT NUMBER (Nu) AT y = -1

| G | I | II | III | IV | V | VI | VII | VIII |
|--------------------------|----------|-----------|------------|-----------|----------|-----------|------------|-------------|
| 1x10³ | 7.27155 | 4.32530 | 2.33467 | 3.76095 | 4.01308 | -3.41385 | -7.40852 | -11.92222 |
| 2x10³ | 27.35626 | 14.65595 | 6.29995 | 10.95150 | 11.76787 | -13.22510 | -27.44243 | -44.18264 |
| -1x10³ | 8.75461 | 4.85243 | 2.45550 | 4.47948 | 6.22891 | -3.10398 | -6.96913 | -11.76807 |
| -2x10³ | 30.32238 | 15.71022 | 6.54162 | 12.38857 | 16.19952 | -12.60537 | -26.56363 | -43.87434 |
| M | 2 | 4 | 6 | 6 | 6 | 6 | 6 | 6 |
| α | 2 | 2 | 2 | 4 | 6 | -2 | -4 | -6 |

TABLE – 11 NUSSELT NUMBER (Nu) AT y = -1

| G | I | II | III | IV | V | VI | VII | VIII | IX |
|--------------------------|------------|------------|------------|-----------|-----------|------------|------------|-------------|------------|
| 1x10³ | -1.14483 | 5.72419 | 7.56530 | 7.76054 | 7.49249 | 2.31337 | 2.29139 | 2.36105 | 2.34872 |
| 2x10³ | -4.54380 | 19.45393 | 28.46432 | 30.41357 | 31.70227 | 6.25400 | 6.20737 | 6.35316 | 6.32830 |
| -1x10³ | -1.26145 | 7.00863 | 11.75362 | 13.56762 | 16.33925 | 2.48350 | 2.51082 | 2.42823 | 2.44094 |
| -2x10³ | -4.77702 | 22.02280 | 36.84095 | 42.02772 | 49.39579 | 6.59427 | 6.64623 | 6.48752 | 6.51274 |
| N | 0.5 | 1.5 | 3.5 | 5 | 10 | 0.5 | 0.5 | 0.5 | 0.5 |
| γ | 2 | 2 | 2 | 2 | 2 | 4 | 6 | 2 | 2 |
| γ₁ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0.5 | 1.2 |

TABLE – 12 NUSSELT NUMBER (Nu) AT y = -1

| G | I | I | III | IV | V | VII |
|--------------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1x10³ | 1.06442 | -1.14483 | 13.70356 | 19.12037 | 2.97572 | 3.93731 |
| 2x10³ | 1.12026 | -4.54380 | 52.79786 | 74.96937 | 8.92364 | 12.85918 |
| -1x10³ | 1.06612 | -1.26145 | 14.89490 | 20.82228 | 3.15698 | 4.20919 |
| -2x10³ | 1.12367 | -4.77702 | 55.18053 | 78.37319 | 9.28615 | 13.40294 |
| P | 0.01 | 0.71 | 7 | 10 | 0.71 | 0.71 |
| Ec | 0.004 | 0.004 | 0.004 | 0.004 | 0.006 | 0.009 |

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