

GEOMETRIC NONLINEAR FINITE ELEMENT ANALYSIS OF SCHWEDLER'S DOME

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ABSTRACT: This paper presents results of Geometric Nonlinear Finite Element Analysis of a Schwedler's dome undergoing snap through instability. The dome is modelled and analysed in ANSYS 12. Non-linear buckling analysis is usually the more accurate approach and is recommended for design or evaluation of actual structure. This technique employs a non-linear static analysis with gradually increasing loads to seek the load level at which the structure becomes unstable. While plotting the equilibrium plot of the analysed structure, the initial mode of deformation for gradual increase in load reaches the shape where the stiffness is lost completely i.e., the nonlinear instability region which "snapthrough" occurs. It is said that the load deflection equilibrium path of the structure has reached a limit point and a dynamic jump occurs to a highly deformed configuration. This process is referred to as snap through or snap buckling. Snap through results in total failure. A parametric study for dome is conducted for the dome analysed above by increasing the height between the rings and also by changing the radius of rings to study the change in the behaviour of dome. The stress in members of the dome is calculated for 50 kN load. The maximum stress is observed in the top ring and top rib. The ribs of the dome are undergoing compression and rings are undergoing tension. But after buckling the configuration of dome is changed and top rib is undergoing tension and top ring is undergoing compression.

Keywords: Schwedler's Dome, Static Analysis, Geometric Nonlinearity, Snap through Buckling.

1. INTRODUCTION

1.1 General

Structural systems, which enable the designers to cover large spans, have always been popular during the history. Beginning with the worship places in the early times, sports stadia, assembly halls, exhibition centres, swimming pools, shopping centres and industrial buildings have been the typical examples of structures with large unobstructed areas nowadays. Dome structures are the most preferred type of large spanned structures. Domes have been of a special interest in the sense that they enclose a maximum amount of space with a minimum surface. This feature provides economy in terms of consumption of constructional materials.

W. Schwedler, a German Engineer, who introduced Schwedler dome in 1863, built numerous braced domes during his life time. A Schwedler dome, one of the most popular types of dome, consists of meridional ribs connected together to a number of horizontal polygonal rings. To stiffen the resulting structure so that it will be able to resist unsymmetric loads, each trapezium formed by intersecting meridional ribs with horizontal rings is subdivided into two triangles by introducing a diagonal member.

Many attempts have been made in the past to simplify the analysis of Schwedler domes, but it is only during the last decade that precise methods of analysis using computers have finally been applied to find the actual stress distribution in these structures.

1.2 Behaviour of Domes

A shell dome resists loads with a force system acting in the surface of the shell. Typically, there will be a principal compressive force acting vertically in the surface of the dome and a lesser horizontal force (usually tensile) acting around the dome.

The way a braced dome works depends on the configuration of the members. Braced domes which are fully triangulated will have a high stiffness in all directions in the surface of the dome. These configurations are also kinematically stable (no mechanism) when idealized as a space truss. Accordingly, the forces in a fully triangulated dome will be principally axial and will have direction and magnitude similar to those in a

shell dome.

1.3 Nonlinear Problems of Trusses

Plane and space trusses are widely used in various types of structures such as bridges and towers, roofs of industrial buildings and sport stadia, exhibition halls and airport hangers. The main advantages of these structures are that they are light in weight, have a high degree of indeterminacy and great stiffness, simple production and fast assembly, totally prefabricated, do not need site welding. They are easily formed into various attractive geometrical surfaces and have the ability to cover large areas with widely spaced column supports. Trusses have a good response against earthquakes and are cost effective. Material nonlinearity and geometric nonlinearity are two different types of nonlinearities that a truss structure may face.

1.3.1 Material Nonlinearity

The material nonlinearity arises when the material properties become functions of state of stress or strain. Nonlinear elasticity, plasticity, viscoelasticity, Creep, or inelastic rate effects are some of the examples which belong to this category.

1.3.2 Geometric Nonlinearity

The geometric nonlinearity arises when the deformations are large enough to alter the equilibrium of the structure. So the equilibrium equations must be written with respect to the deformed structural geometry. In addition loads may change their direction as the deformation increases. Slender structures in aerospace, civil and mechanical engineering applications, tensile structures such as cables, stability analyses of all types are some of the examples in this category.

1.4 Structural Instability

The structure equilibrium state must be stable, but due to nonlinearity, the structure may suffer the problem of loss of stability. In a practical sense, an equilibrium state of a structure or a system is said to be stable if accidental forces, shocks, vibrations, eccentricities, imperfections, inhomogeneities or other probable irregularities do not cause the system to depart excessively or disastrously from the state.

1.4.1 Buckling

A structure may fail due to the stresses exceeding the given safe limits or the loss of stability due to tensile loads when the material becomes nonlinear. On the other hand, in the case of geometric nonlinearity, the structure may fail due to inability to keep up the form. The loss of stability under compressive loads is usually termed structural (or geometrical or form) instability, commonly known as buckling.

In geometric instability problems there is seen a change in the geometry or position of the system due to the appearance of the characteristic displacement. The geometry change in the system, a consideration of which is one of the typical features of structural stability analyses is a reason for either introducing additional new forces or changing the nature of the forces that existed prior to the loaded position. In terms of these new forces which appear during the loss of structural stability, there can be further classification of instability as follows.

- a) Flexural buckling
- b) Torsional buckling
- c) Torsional – flexural buckling
- d) Snap-through buckling

To understand the different buckling phenomena, consider a centrally loaded column, with the cross section in the form of an I-section, subjected to external disturbances in the form of displacement in the plane of the web. In addition to the shear on the web there will be bending also due to these external disturbances. The loss of stability in this case is by flexural buckling in the plane of the web. However if the external disturbances in the form of displacement is given in such a way that the flanges bend in their respective planes but in opposite direction, the column will be subjected to torsion in addition to compression, then the loss of stability is by torsional buckling. Consider the same column subjected to eccentric loading with no lateral support to its compression flange. Even though the column deflects initially in the plane of the web but as load keeps increasing, the column may fail due to loss of stability in combined mode of twist and lateral buckling of the cross section. The compression flange which is unstable tends to buckle laterally due to external disturbances whereas the tension flange which is stable tends to remain straight. Here the loss of stability is due to torsional-flexural buckling also known as lateral buckling.

In snap-through buckling the loss of stability is due to transition to a non-proximate equilibrium configuration with a sudden change in the nature of internal forces of the structure. Non-linear buckling analysis is usually the more accurate approach and is recommended for design or evaluation of actual structure. This technique employs a non-linear static analysis with gradually increasing loads to seek the load level at which your

structure becomes unstable. To summarize, one major characteristic of non-linear buckling, as opposed to eigen value buckling, is that non-linear buckling phenomenon includes a region of instability in the post-buckling region, whereas eigen value buckling only involves linear, pre-buckling behaviour up to the bifurcation (critical loading) point.

1.5 Analysis of Space Trusses

In architecture and structural engineering, a truss (or braced framework) is formed when members are put and connected together at joints (or nodes). It is comprised by one or more triangular units together with straight slender members. Truss analysis can be carried out by assuming the truss as a planar truss or space truss. Planar truss or Plane truss is a truss where all the members and nodes are lying within a two-dimensional plane. Space truss is a truss that having members and nodes extending into three dimension. In most cases in the analysis of truss, a truss is usually modelled and analysed as a two-dimensional plane frame. At the same time, if there is significant forces out-of-plane, the truss should be modelled as a three-dimensional space for further analysis.

1.5.1 Finite Element Method

The finite element method sometimes also referred to as finite element analysis (FEA), is a computational technique used to obtain approximate solutions of boundary value problems in engineering. Boundary value problem is also called field problems, which is the domain of interest and most often represents a physical structure. Simply stated, it is a mathematical problem in which one or more dependent variables must satisfy specific conditions on the boundary of the domain. Normally, the field variables are the dependent variables of interest governed by the differential equation. Depending of the type of physical problem being analyzed, the field variables may include physical displacement, temperature, heat flux, and fluid velocity.

1.5.2 Finite Element Analysis of Nonlinear Problems

Linear static analysis of structures deals with static problems in which the structural response is linear in the cause-and-effect sense. For example, doubling the applied forces doubles the displacements and internal stresses. Problems outside this domain are classified as nonlinear.

It is straight forward to solve linear problems and finite element methods presently dominate the scene, whereas for nonlinear problems still the dominance of finite element methods continues but it is tricky. Incremental schemes have to be used to apply the load and iterative schemes like Newton-Rapson, modified Newton-Rapson, should be used within increment with proper convergence criteria at a desired tolerance limit to get the converged and acceptable solution for a particular step.

2. FINITE ELEMENT ANALYSIS

2.1 General

Finite Element Analysis (FEA) is a computer simulation technique used in engineering analysis by using the numerical technique of finite element method (FEM). Mechanical engineering software widely used for such analysis is ANSYS 12.

2.2 Description of Software

ANSYS structural mechanics offers wide range of analysis from concept simulation to advanced analysis. With a full complement of linear and nonlinear elements, material laws ranging from metal to rubber and the most comprehensive set of solvers available, ANSYS simulation tools are applied widely by users across industries. Additionally, the adaptive architecture of ANSYS software tools provides one with the flexibility for customization and interoperability with other tools such as third-party software.

2.3 Element Used

The element used for the modelling the dome is Link 180. Link180 is a spar that can be used in a variety of engineering applications. This element can be used to model trusses, sagging cables, links, springs, etc. This 3-D spar element is a uniaxial tension-compression element with three degrees of freedom at each node: translations in the nodal x, y, and z directions. Tension-only (cable) and compression-only (gap) options are supported. As in a pin-jointed structure, no bending of the element is considered. Plasticity, creep, rotation, large deflection, and large strain capabilities are included.

2.4 Schwedler's Dome

The geometry of dome consists of 264 truss elements as shown in figure 2. This example is seen in literature related to non linear finite element analysis of space trusses (M. Greco et al 2005)

Area of members, $A = 3200 \text{ mm}^2$

Modulus of elasticity, $E = 200000 \text{ N/mm}^2$

Poisson's ratio = 0.3

Schwedler's Dome top and front input data are shown in figure 1.

3. GEOMETRIC NONLINEAR FINITE ELEMENT ANALYSIS OF SCHWEDLER'S DOME

Geometric nonlinear FEA is done with ANSYS 12 for the Schwedler's dome. When significant changes in stiffness occurs the load-deflection curve becomes nonlinear. The challenge is to calculate the nonlinear displacement response using a linear set of equations. One approach is to apply the load gradually by dividing it into a series of increments and adjusting the stiffness matrix at the end of each increment. The dome is modelled in ANSYS 12. The link 180 element is used. Bottom ring is considered as fixed. Material is assumed to be homogenous linear isotropic. Load of 50 kN applied as increments at the central node at the top of the crown of the dome. The nonlinear analysis is done for the dome and the maximum deflection is found as 749 mm.

The equilibrium plot of the above analysed structure is plotted for central node in Figure 2. The initial mode of deformation for gradual increase in load reaches the shape where the stiffness is lost completely i.e., the nonlinear instability region which "snapthrough" occurs. It is said that the load deflection equilibrium path of the structure has reached a limit point and a dynamic jump occurs to a highly deformed configuration. This process is referred as snap through or snap buckling. Snap through results in total failure. The buckling load for this dome is observed as 29.895 kN.

3.1 Parametric Study by Increasing the Height between the Rings

A parametric study for dome is conducted for the dome analysed above by increasing the height between the rings to study the change in the behaviour of dome. Dome is modelled by increasing the height between the rings by 0.5, 1 and 1.5 metre. Load of 50 kN applied as increments at the central node at the top of the crown of the dome and deflection of dome is found in each case. For 0.5m height increase, the deflection is found to be 735 mm and for 1m height increase the deflection is reduced to 723 mm. The deflection of dome is found as 717 mm for 1.5m height increase. Variation of buckling load corresponding to increase in height is shown in table 2 and graph plotted in figure 3. By increasing the height, the buckling load is found to be reducing considerably. It is said that buckling load of the dome depends on the height to span ratio of dome.

3.2 Parametric Study by Increasing the Radius

A parametric study for dome is conducted for the dome analysed above also by changing the radius of rings to study the change in the behaviour of dome. Dome is modelled by increasing the radius of the rings by 0.2, 0.4 and 0.6 metre and Load of 50 kN applied as increments at the central node at the top of the crown of the dome and deflection of dome is found in each case. For 0.2m radius increase, the deflection is found to be 755mm and for 0.4m radius increase, the deflection is increased to 759mm. The deflection of dome is found as 766mm for 0.6m radius increase. The deflection is found increasing by increasing the radius of dome. Buckling loads are found in each model and variation of buckling load corresponding to increase in radius is shown in table 3 and graph plotted in figure 4. By increasing the radius, the buckling load is found reducing considerably while keeping the height constant.

3.3 Stress Variation In Rings And Ribs

Stress variation in rings and ribs are also found in all the models which are tabulated in table 1. The stress in members of the dome is calculated for 50 kN load. The maximum stress is observed in the top ring and top rib. The ribs of the dome are undergoing compression and rings are undergoing tension. But after buckling the configuration of dome is changed and top rib is undergoing tension and top ring is undergoing compression.

4. CONCLUSION

1. Geometric Nonlinear Finite Element Analysis of Schwedler's dome is done for a force applied at the centre node at the crown. Equilibrium plot is observed for the dome while load is applied as increments. It is found that when it reaches a critical load a sudden snap is observed and the structure changes to a highly deformed configuration.

2. Parametric study by increasing the height is done for the Schwedler's dome by keeping the radius constant. By increasing the height of dome buckling load is found to be decreasing considerably. A small decrease in snap is also observed by increasing the height.

3. A parametric study is carried out by changing the radius of the dome by keeping the height constant. It is seen that the buckling load is found to be decreasing by increasing the radius. There is only very slight decrease in the snap for increase the radius.

4. The stress in various members is found. It is seen that the rings are subjected to tension and ribs are subjected to compression for smaller loads. But after buckling the structure lost its original configuration changes to another stable configuration and the top rib is experiencing tension and top ring is experiencing compression.

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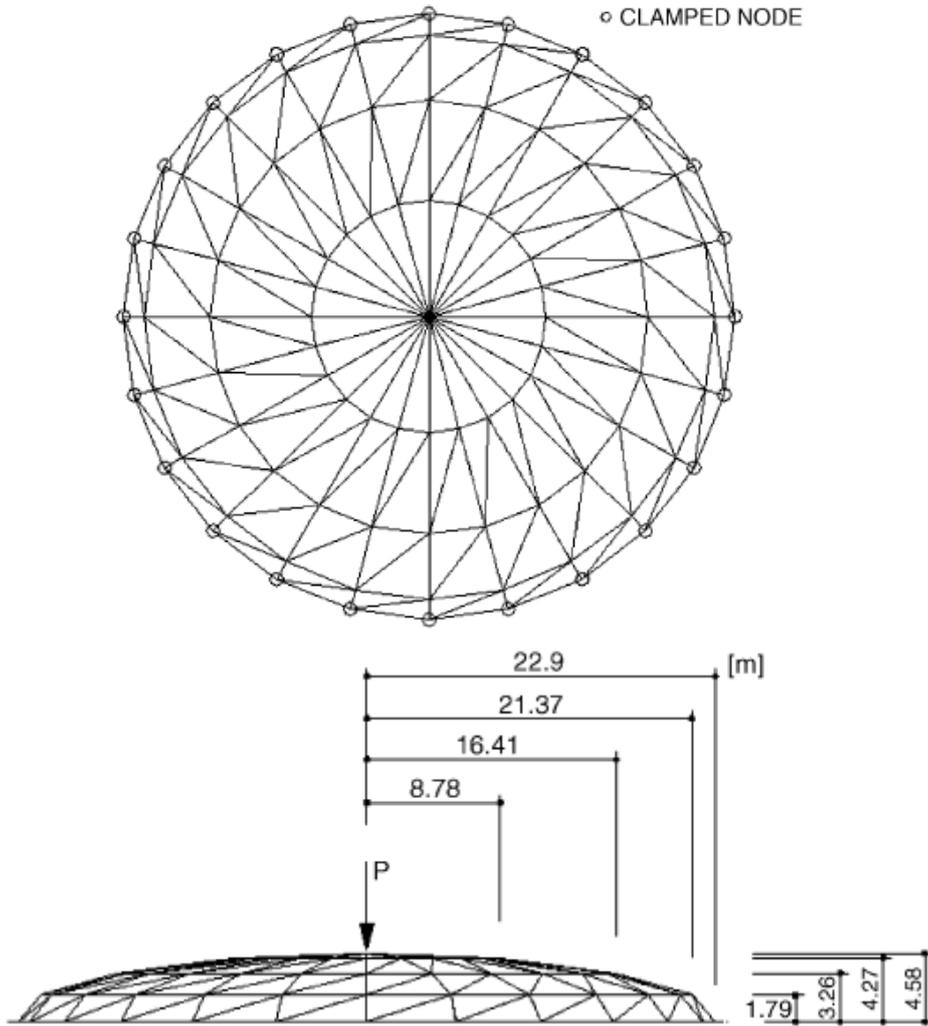


Figure.1:Schwedler's Dome Top and Front Input Data

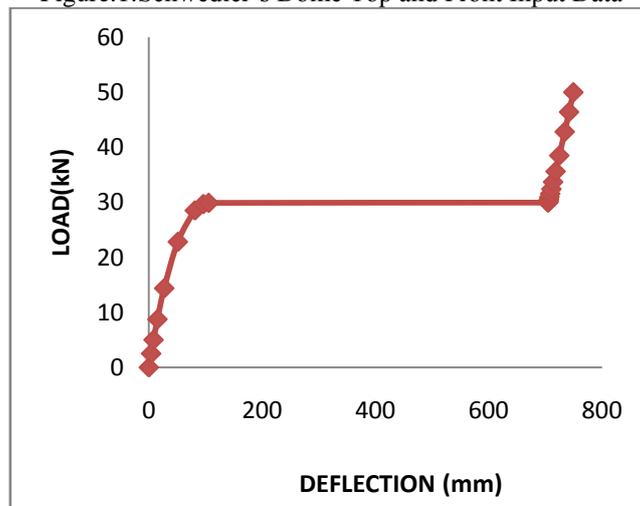


Fig.2: Load Deflection Curve

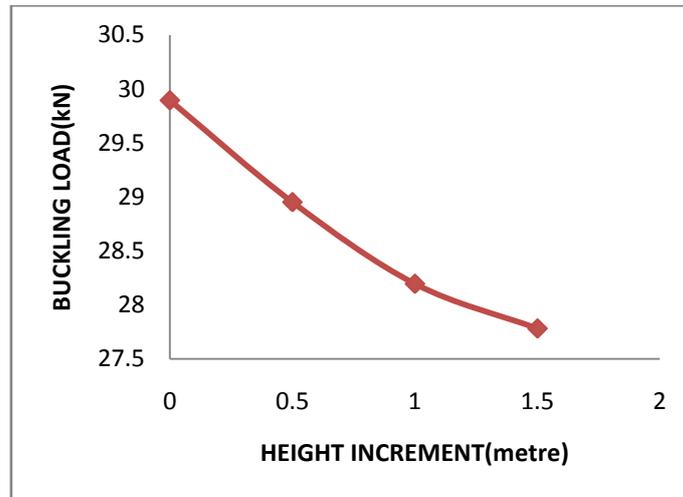


Fig.3: Buckling Load versus Height Increment Plot

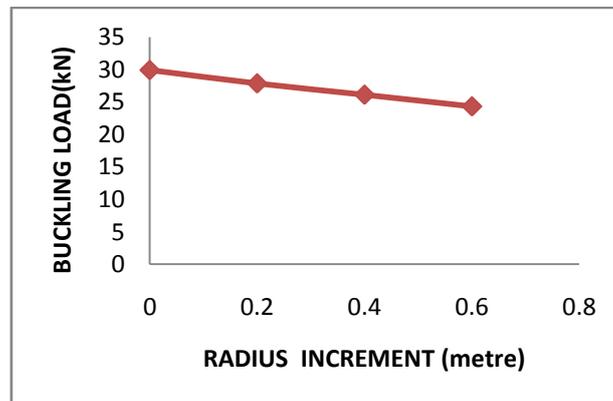


Fig.4: Buckling Load versus Radius Increment Plot

Table.1 Stress in members

Element Type	Dome Having Height 4.58 Metre And Radius 22.9 Metre	Increasing The Height of The Dome			Increasing The Radius of The Dome		
		Increasing The Height by 0.5 Metre	Increasing The Height by 1 Metre	Increasing The Height by 1.5 Metre	Increasing The Radius by 0.2 Metre	Increasing The Radius by 0.4 Metre	Increasing The Radius by 0.6 Metre
	Stress (N/mm ²)	Stress (N/mm ²)	Stress (N/mm ²)	Stress (N/mm ²)	Stress (N/mm ²)	Stress (N/mm ²)	Stress (N/mm ²)
Rib 1	14.149	14.53	14.46	14.29	14.352	14.421	14.65
Rib 2	-5.17	-2.275	-2.868	-3.46	-5.18	-4.493	-5.19
Rib 3	-2.308	-1.27	-1.47	-1.67	-2.308	-2.3	-2.308
Rib 4	-0.859	-0.68	-0.719	-0.74	-0.859	-0.824	-0.859
Ring 1	-73.9	-63.93	-66.008	-68.386	-74.761	-75.76	-75.76
Ring 2	11.19	4.1504	5.645	7.14	11.214	11.04	11.252
Ring 3	6.335	3.397	3.881	4.365	6.335	6.47	6.334

Table.2 Change In Buckling Load With Height Increment

Height Increment(Metres)	Buckling Load Of Dome (kN)
0	29.895
0.5	28.952
1	28.196
1.5	27.782

Table.3 Change In Buckling Load With Radius Increment

Radius Increment (Metre)	Buckling Load of Dome (kN)
0	29.895
0.2	27.840
0.4	26.099
0.6	24.304