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Modeling the Dual Long Memory in the Moroccan Stock Market: ARFIMA-FIGARCH Approach

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Abstract: The purpose of this paper is to test the weak form of the informational efficiency hypothesis on the Moroccan stock market during the period from 03/01/2002 to 15/08/2023, by examining the presence of dual long memory in both returns and volatility of the Casablanca Stock Exchange index (*MASI*). We initiated our study by testing the random walk hypothesis using various standard statistical tests, such as the normality test, stationarity tests, return autocorrelation tests, and the variance ratio test. The results of these tests strongly rejected the random walk hypothesis for the Moroccan stock market over the examined period, thus concluding that the Casablanca Stock Exchange is not an efficient market in its weak form.

Subsequently, we tested the presence of dual long memory in both the conditional mean and conditional variance of the geometric returns of the *MASI* index by applying four joint models: *ARFIMA-FIGARCH*, *ARFIMA-FIEGARCH*, *ARFIMA-FIAPARCH*, and *ARFIMA-HYGARCH*. Various combinations of the parameters for these four models were tested, and we selected the models with the most significant estimations. Empirical results from all these models indicated that both long memory parameters are statistically significant at the 1% or 5% significance level, and most of the other parameters are statistically significant, except occasionally 1 or 2 parameters. These findings robustly confirm that the Moroccan stock market is inefficient in its weak form.

Keywords: Informational efficiency, autocorrelation test, variance test, *ARFIMA*, *FIGARCH*, *FIEGARCH*, *FIAPARCH*, *HYGARCH*

JEL classification: C01, C13, C22, G14

1. Introduction

1.1 Efficient Market Hypothesis

The concept of informational efficiency in financial markets, essential to economics and finance, refers to the speed and accuracy with which markets incorporate available information into the prices of financial assets. It is based on the idea that in efficient markets, asset prices reflect all relevant information at the moment, whether it is public or private. This concept of informational efficiency was initially formulated by Eugene Fama in the 1960s (Fama (1965)) in the form of the Efficient Market Hypothesis (*EMH*). The history of the Efficient Market Hypothesis (*EMH*) can be divided into two stages. The first stage involves the construction of the theory in the 1960s. In the second stage, the establishment of empirical confirmation made the theory consensus in the 1970s (Fama (1970).

Eugene Famahas decomposed efficiency into three main forms:

- **Weak form efficiency:** This form states that financial asset prices already reflect all historical data, making technical analysis, based on price and volume histories, ineffective for predicting future price movements.
- **Semi-strong form efficiency:** This form asserts that financial asset prices not only reflect historical data but also all publicly available information, including company financial data, economic and political news, as well as company announcements, among others. Therefore, neither fundamental nor technical analysis should enable obtaining exceptional returns.
- **Strong form efficiency:** This form posits that financial asset prices reflect not only all historical and public data but also all private information, including information accessible only to insiders.

1.2 Importance of studying informational efficiency in financial markets

Studying the informational efficiency of financial markets, especially in its weak form, is a fundamental concept with profound implications in the financial domain:

- **Efficient resource allocation:** In an efficient market, the allocation of financial resources is expected to be more effective, enabling investors to make informed decisions.
- **Investment and portfolio management:** Individual and institutional investors rely on informational efficiency to make informed investment decisions. In the case of market inefficiency, increased opportunities to generate abnormal returns may emerge, impacting portfolio management strategies.
- **Financial risk management:** Informational efficiency provides investors with the opportunity to assess the inherent risks in their investments with increased accuracy.
- **Reduction of information asymmetry:** Informational efficiency mitigates the inequality of access to information among various participants in financial markets.
- **Financial market stability:** Mastering the concept of informational efficiency is essential for regulatory bodies to preserve the stability of financial markets.
- **Financial innovation:** Informational efficiency can spur innovative impulses within the financial domain. For instance, new technologies and analytical data models can be developed to more effectively leverage financial information and optimize risk management.
- **Fraud prevention:** Regulatory bodies use the concept of informational efficiency to identify fraudulent practices within financial markets.

1.3 Problem statement

In this study, we address two interconnected questions. Firstly, how can we capture long memory in the financial series of the Moroccan stock market index? Subsequently, the answer to this initial question determines the second one: Is the Moroccan stock market efficient?

To address the first question, we applied several joint models analyzing the dual long memory property in both the conditional mean and conditional variance of the MASI index. We estimated four joint models: *ARFIMA-FIGARCH*, *ARFIMA-FIEGARCH*, *ARFIMA-FIAPARCH*, and *ARFIMA-HYGARCH*, under different distribution assumptions such as Normal distribution, Student's distribution, Skewed Student's distribution, and Generalized Error Distribution (*GED*).

These joint models combine two components: the *ARFIMA* model (Autoregressive Fractionally Integrated Moving Average), which captures long-term dependencies (long memory) in the returns of the *MASI* index, and the *FIGARCH* (Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity), *FIEGARCH* (Fractionally Integrated Exponential *GARCH*), *FIAPARCH* (Fractionally Integrated Asymmetric Power *ARCH*), or *HYGARCH* (Hyperbolic *GARCH*) models, which capture the long-term memory of the conditional volatility of this index.

The method employed to address the primary inquiry serves as a foundation for addressing the subsequent question related to the effectiveness of the Moroccan stock market. By examining long-term memory patterns in returns and volatility, it becomes possible to determine the extent to which the market efficiently incorporates information and responds to new data, thereby establishing the level of efficiency of the Moroccan stock market.

2. Literature Review

Extensive research has been undertaken worldwide to scrutinize the empirical evidence of the Efficient Market Hypothesis (*EMH*).

Grossman and Stiglitz (1980) suggested that the key to achieving informational efficiency in the market lies in recognizing that a higher number of well-informed investors leads to a more informative price system. The balance between well-informed and poorly informed individuals in the economy depends on various constraints such as the cost, quality, availability, and timing of information. Similarly, Fama (1991) argued that lower transaction costs in a market make it more efficient.

To assess the weak form of market efficiency or the random walk hypothesis, various statistical tools have been employed, including serial correlation, runs test, variance ratio test, multiple variance ratio test, spectral analysis, unit root tests, *ARMA*, *ARIMA*, *ARCH*, and *GARCH* models.

Extensive research has been conducted worldwide to collect empirical evidence of *EMH*. Most studies on stock markets in developed countries such as France, the United States, Japan, and the United Kingdom appear to validate the weak form efficiency hypothesis. In contrast, test results on markets in emerging and frontier countries have generally led to the rejection of the random walk hypothesis.

In this study, our focus is on the Moroccan stock market, characterized as a frontier (pre-emergent) market due to its low market capitalization and limited liquidity. Frontier markets hold a special interest as they offer a greater potential for growth compared to developed markets.

In this literature review, we will first discuss some research on the efficiency hypothesis in stock markets of developed, emerging, and frontier countries that used simple statistical methods to test the random walk of returns in these markets. Next, we will review studies relevant to our research that have investigated the utilization of techniques designed to capture the prolonged influence of returns and volatilities.

Studies testing the Random Walk Hypothesis

Several studies have been conducted to test the random walk hypothesis in the stock markets of Europe and the Americas. One notable study by Urrutia (1995) utilized the runs test and the variance ratio test to examine the random walk hypothesis on the monthly returns of Latin American emerging markets indices from December 1975 to March 1991. The runs test indicated efficiency in the Latin American stock markets under the weak form, while the variance ratio test rejected the random walk hypothesis.

Similarly, Borges (2010) conducted a study to test the weak form of *EMH* for stock indices in the United Kingdom, France, Germany, Spain, Greece, and Portugal. Using a combination of simulation tests and the joint variance ratio test with daily and weekly data from 1993 to 2007, the study presented nuanced conclusions. It refuted EMH for daily data in Portugal and Greece due to positive first-order autocorrelation in returns. Weekly data for France and the UK also rejected *EMH* due to mean reversion. However, tests for Germany and Spain did not refute *EMH*.

In another study focusing on European stock markets, Dutta (2015) employed the runs test, variance ratio test, and unit root tests (Dickey-Fuller augmented and Phillips-Perron) to investigate weak form efficiency in France, Germany, Italy, and the UK. The tests were applied to monthly price index data from 1998 to 2014. The empirical results suggested that sampled European stock markets did not follow a random walk, indicating weak form inefficiency.

To summarize, there is still no consensus in the existing literature on the efficiency or inefficiency of European and American stock markets.

As for studies on the weak form efficiency of emerging and frontier stock markets, Mollah (2007) tested the weak form efficiency of the Botswana Stock Exchange from 1989 to 2005, using daily return series and applying the runs test, autocorrelation test, and *ARIMA* model. Empirical evidence from the tests rejected the random walk hypothesis, concluding that the Botswana Stock Exchange is not weak-form efficient.

Al-Jafari and Altaee (2011) examined the efficiency of the Egyptian stock market using various statistical tests. They analyzed the daily prices of the EGX 30 index from January 1998 to December 2010. Empirical results contradicted the random walk hypothesis and weak efficiency of the Egyptian stock market.

Büyükşalvarcı and Abdioğlu (2011) conducted a study to evaluate the weak form efficiency of the Istanbul Stock Exchange (ISE) in Turkey. Analyzing daily data from various ISE indices from October 23, 1987, to July 15, 2011, they applied parametric tests like the augmented Dickey Fuller unit root test, serial autocorrelation test, and variance ratio test, as well as non-parametric tests including the Phillips-Perron unit root test and the runs test. Except for the runs test results for service and technology indices, the runs test, autocorrelation test, and variance ratio test results rejected the random walk hypothesis for the Turkish stock market.

Gimba (2012) tested the weak form efficiency of the Nigerian Stock Exchange (NSE). Analyzing daily and weekly data for the Nigerian stock index and the five most traded and oldest bank stocks from January 2007 to December 2009, empirical results from autocorrelation tests convincingly rejected the null hypothesis of a random walk for the stock index and four of the five bank stocks studied. Overall, the author firmly established the inefficiency of the NSE under weak form efficiency.

Similarly, McKerrow (2013) analyzed random walk models in the emerging stock markets of Botswana, Côte d'Ivoire, Ghana, Mauritius, and Namibia. Using monthly series data covering about 16 years, employing the random walk method, and conducting the runs test and multiple variance ratio test, the study's conclusions were mixed. The runs test indicated rejection of the random walk hypothesis for Namibia and Côte d'Ivoire markets but acceptance for Botswana, Ghana, and Mauritius markets.

Jaihan and Syed (2013). (2013) tested the weak form of the efficient market hypothesis on the Karachi Stock Exchange (KSE) using daily data from 2006 to 2011. Three tests, namely the augmented Dickey-Fuller unit root test, runs test, and autocorrelation test, were applied to the data. The study revealed that the KSE is weakly inefficient and non-random.

Phan and Zhou (2014) conducted an investigation to test the weak form efficiency hypothesis in the Vietnamese stock market. Employing three distinct statistical methods-autocorrelation test, variance ratio test, and runs test—on weekly returns from July 28, 2000, to July 28, 2013, the study provided strong evidence against the random walk hypothesis over the examined period.

Chiny and Mir (2015) conducted an in-depth study on the weak efficiency of the Moroccan stock market. They subjected the daily returns of four stock indices - the Casablanca Stock Exchange index (MASI), the banking sector index (BNQ), the insurance sector index (ASSUR), and the real estate sector index (IMMO) - to autocorrelation testing, unit root testing, variance ratio testing, and Runs testing. This analysis covered the period from January 1, 2002, to December 31, 2013. The results obtained from these various tests unequivocally rejected the weak form efficiency hypothesis for the four markets.

Studies testing the presence of long memory

Several studies have investigated informational efficiency by examining the long memory of stock market indices using models such as *ARFIMA* and *FIGARCH*.

Among these studies, we mention the works of Lamouchi (2020), Falloul (2020), and Ziky and Ouali (2021), who attempted to capture long memory in the return series of the stock index using the *ARFIMA* model.

Lamouchi (2020) applied the *ARFIMA* model to the Tadawul index of the Saudi stock market to capture the long memory of daily returns from 1998 to 2020. The results show that the Saudi stock market exhibits long-term memory, contradicting the Efficient Market Hypothesis (*EMH*).

A similar study was conducted by Falloul (2020) to test the weak form of the efficiency hypothesis of the Moroccan stock market, applying the *ARFIMA* model to the daily returns of the MASI index. The study revealed that the Moroccan stock market exhibits long memory, rejecting the efficiency hypothesis of this market.

Similarly, Ziky and Ouali (2021) tested the efficiency of the Moroccan stock market in its weak form using the ARFIMA model to capture long memory in the daily return series of the MASI index from 1992 to 2016. The results indicate that the Moroccan stock market is characterized by long memory and can be considered inefficient.

Other researchers have attempted to capture long memory in the volatility of stock market indices using the *FIGARCH* model family. Among these researchers, Chaker A. (2003) examined the long memory property in the volatility of the Tunisian stock market through daily data for two indices, IBVMT and TUNINDEX, from 1998 to 2004, applying the FIGARCH model. This study demonstrated that the volatility of the Tunisian stock market exhibits long memory, which is inconsistent with the market efficiency hypothesis.

Similarly, Maheshchandra (2014) examined the existence of long memory in the volatility of daily returns of stock market indices in India (BSE) and China (SSE) from January 1, 2009, to June 24, 2014, using the FIGARCH model. The results indicate strong evidence of long-term memory in the conditional variance of stock market indices. The long-term memory property of the BSE Indian stock market is revealed to be stronger than that of the Chinese SSE stock market.

Alfred and Sivarajasingham (2020) tested the efficiency hypothesis of the daily return series of the Sri Lankan stock index from January 2, 1985, to September 28, 2018. They applied the *ARFIMA* model to capture long memory in return series and the FIGARCH model to capture long memory in the conditional volatility of the Sri Lankan index. The results show that the return series does not have long memory, while the volatility series exhibits long memory.

Another category of studies has examined the presence of long memory in both stock index returns and their conditional volatilities, applying separately the *ARFIMA* and *FIGARCH* (or *FIEARCH*) models.

Among these studies, we cite Kang and Yoon (2006), who examined the presence of long memory in the returns of indices from four Asian stock markets - Japan, South Korea, Hong Kong, and Singapore - applying the *ARFIMA* model, and analyzed the existence of long-term memory in the conditional variances of these four indices, applying the FIEGARCH model. The results of the *ARFIMA* model revealed no evidence of long memory in the returns of the four indices, while the results of the *FIEGARCH* model detected long memory in the volatilities of the four indices.

In a similar study, Nazarian et al. (2014) applied the *ARFIMA* model to capture long-term memory in the returns of the Tehran Stock Exchange (TSE) index and the FIGARCH model to detect long memory in the conditional variance (volatility) of the TSE index. The results of the *ARFIMA* model indicate the absence of long memory in the return series of the TSE index, while the results of the *FIGARCH* model show evidence of long memory in the conditional variance of this series.

Finally, a last category of studies simultaneously analyzed the presence of long memory in both stock index returns and volatilities, applying the joint *ARFIMA-FIGARCH* model.

Among these studies, we mention the study of Turkyilmaz and Balibey (2014), who examined the weak form efficiency of the Karachi stock market in Pakistan for the period 2010-2013, applying the *ARFIMA-FIGARCH* model. According to the results of the study, the *ARFIMA* component of the model does not support long-term memory behavior for the returns of the Karachi market, while the *FIGARCH* component of the model indicates that the volatility of the market's returns exhibits long memory.

A similar study was conducted by Mahboob et al. (2017) to explore the presence of long memory in the daily returns and volatilities of the Dhaka Stock Exchange index in Bangladesh over the period from December 15, 2003, to July 31, 2013, applying both ARFIMA-FIGARCH and ARFIMA-FIPARCH models. The test results clearly indicate the existence of long memory in both returns and volatilities in the Dhaka stock market.

Another study by Houfi (2019) aimed to test the weak form of informational efficiency of the Tunisian stock exchange. The author examined the behavior of long memory in the series of daily returns and volatility of the Tunisian stock index, applying the ARIMA-FIGARCH model. The empirical study covered a sample period from January 2, 1998, to March 16, 2018. The results showed the presence of long memory in both the returns and volatility of the Tunisian stock market.

In a similar context, Bouchareb et al. (2021) applied the ARFIMA-FIGARCH model to capture long memory in both returns and volatilities of four Mediterranean stock markets, namely Morocco, Turkey, Spain, and France, over the period 2000-2020. The results provide strong evidence of long memory in both returns and volatilities for the Moroccan and French stock markets, and only in volatility for the Spanish and Turkish markets, thus rejecting the efficiency hypothesis of these markets.

The presence of long memory was also explored by Odonkor et al. (2022) in the daily returns and volatilities of seven stocks from the Ghana Stock Exchange using the ARFIMA-FIGARCH model. In this study, the authors found that all seven stocks exhibit long memory in both returns and volatility, contradicting the efficiency hypothesis of the Ghanaian stock market.

4. Data and Methodology

4.1 Data

The data utilized in this study consists of the daily closing prices of the Casablanca Stock Exchange index (*MASI*), spanning from 03/01/2002 to 15/08/2023, comprising a total of 5393 observations.

Subsequently, the *MASI* index prices were transformed into geometric returns:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1}) \quad (1)$$

where P_t represents the index price, and ln corresponds to the natural logarithm. The covered period is between 03/01/2002 and 11/08/2023, resulting in 5392 geometric returns.

The data was downloaded from the website www.investing.com.

4.2 Methodology

In this section, we describe the joint models based on the *ARFIMA* model capturing the long memory of returns, and on the *FIGARCH*, *FIEGARCH*, *FIPARCH*, or*HYGARCH* models capturing the long memory of volatility.

Definition of the ARMA($\bar{p}, \bar{q}$) model: AutoRegressive Mobile Average

Let (X_t) be a stationary stochastic process. We say that (X_t) is an $ARMA(\bar{p}, \bar{q})$ process of orders p and q if there exist lag polynomials $\phi(L)$ of order $\bar{p}$ and $\psi(L)$ of order $\bar{q}$ whose roots are all outside the unit circle, and a white noise (ε_t) such that:

$$\phi(L)X_t = c + \psi(L)\varepsilon_t \quad (2)$$

where L is the lag operator, and ϕ and ψ are lag polynomials defined by:

$\phi(L) = 1 - \sum_{i=1}^{\bar{p}} \phi_i L^i \quad \text{avec } \phi_p \neq 0 \qquad\qquad \psi(L) = 1 + \sum_{j=1}^{\bar{q}} \psi_j L^j \quad \text{avec } \psi_q \neq 0$ (3)

The $ARMA(\bar{p}, \bar{q})$ process can also be expressed as:

$$X_t = c + \sum_{i=1}^{\bar{p}} \phi_i X_{t-i} + \sum_{j=0}^{\bar{q}} \psi_j \varepsilon_{t-j} + \varepsilon_t \tag{4}$$

Definition of the ARIMA$(\bar{p}, d, \bar{q})$ model: AutoRegressive Integrated Moving Average

Let (X_t) be a non-stationary integrated stochastic process of order $d \in \mathbb{N}^*$. We say that (X_t) is an $ARIMA(\bar{p}, d, \bar{q})$ process of orders $\bar{p}$, d and $\bar{q}$ if there exist lag polynomials $\phi(L)$ of order $\bar{p}$ and $\psi(L)$ of order $\bar{q}$, with roots all outside the unit circle, and a white noise (ε_t) such that:

$$\phi(L)(1-L)^d X_t = c + \psi(L)\varepsilon_t \tag{5}$$

where L is the lag operator, and ϕ and ψ are defined as previously.

Definition of the ARFIMA$(\bar{p}, d, \bar{q})$model: AutoRegressive Fractionally Integrated Mobile Average

We say that (X_t) is an $ARFIMA(\bar{p}, d, \bar{q})$ process of orders $\bar{p}, d \in \mathbb{Q}$ and$\bar{q}$ if there exist lag polynomials $\phi(L)$ of order $\bar{p}$, $\psi(L)$ of order $\bar{q}$ with all roots outside the unit circle, and white noise(ε_t) such that:

$$\phi(L)(1-L)^d X_t = c + \psi(L)\varepsilon_t \tag{6}$$

where L is the lag operator, and ϕ and ψ are defined as previously:

The filter $(1-L)^d$ can be expressed in the form:

$$(1-L)^d = \sum_{j=0}^{\infty} (-1)^j \binom{d}{j} L^j = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)} L^j \tag{7}$$

where $\binom{d}{j}$ is the binomial coefficient, and $\Gamma(.)$is the gamma function.

Properties of an ARFIMA$(\bar{p}, d, \bar{q})$ process:

Let (X_t)be an $ARFIMA(\bar{p}, d, \bar{q})$ process. Then:

- If $-1/2 < d < 1/2$, the$ARFIMA(\bar{p}, d, \bar{q})$ process is stationary with an autocorrelation function $\rho(k)$ that decreases hyperbolically:

$$\rho(k) \sim C.k^{2d-1}$$

- If $0 < d < 1/2$, and if all the roots of $\phi(L) = 0$ are outside the unit circle, then the $ARFIMA(\bar{p}, d, \bar{q})$process is stationary with long memory. Autocorrelations are positive and decrease hyperbolically towards zero as the lag increases.
- If $-1/2 < d < 0$, the $ARFIMA(\bar{p}, d, \bar{q})$process is stationary and anti-persistent (intermediate persistence). Autocorrelations decrease hyperbolically towards zero, and the spectral density is dominated by high-frequency components (it tends to zero as frequency tends to zero).
- If $d \geq 1/2$, then the $ARFIMA(\bar{p}, d, \bar{q})$ process is non-stationary.
- If $d = 0$, the $ARFIMA(\bar{p}, d, \bar{q})$ process reduces to the standard $ARFIMA(\bar{p}, \bar{q})$process with short memory (where the effect of a random shock fades exponentially over time).
- If $d = 1$, we obtain the $ARIMA(\bar{p}, 1, \bar{q})$ process.

Definition of the ARCH(q) model: Autoregressive Conditional Heteroskedasticity

The $ARCH$ model was proposed by Engle (1982) in the early 1980s to estimate the conditional mean and conditional variance of the UK macroeconomic quarterly inflation series between 1958 and 1977. The $ARCH$ model was introduced to capture time-varying volatility known as heteroskedasticity.

Let (X_t) be a stationary process. Let $\Im_{t-1}$ be the set of past information containing the realized values of all relevant variables up to date $t-1$. We define:

$E(X_t/\Im_{t-1}) = \mu_t \qquad\qquad Var(X_t/\Im_{t-1}) = \sigma_t^2$ (8)

It is said that (X_t) is an $ARMA(\bar{p}, \bar{q})$ process with $ARCH(q)$ errors if the following conditions are met:

1) (X_t) is an $ARMA(\bar{p}, \bar{q})$ process written in the form:

$$X_t = c + \sum_{i=1}^{\bar{p}} \psi_{t-i} . X_{t-i} + \sum_{j=1}^{\bar{q}} \phi_{t-j} . \varepsilon_{t-j} + \varepsilon_t \tag{9}$$

where c is a constant, ψ_i and ϕ_j are the parameters of the $ARMA(\bar{p},\bar{q})$ model, and ε_t is the error following an i.i.d. process satisfying:

$$E(\varepsilon_t)=0 \qquad Var(\varepsilon_t)=\sigma^2 \qquad cov(\varepsilon_t,\varepsilon_s)=0 \text{for } t\neq s \tag{10}$$

2) The error ε_t satisfies:

$$E(\varepsilon_t/\Im_{t-1})=0 \qquad Var(\varepsilon_t/\Im_{t-1})=h_t^2 \qquad \varepsilon_t=h_t.Z_t \tag{11}$$

with $h_t>0$ and Z_t a white noise such that $Z_t \rightsquigarrow \mathcal{N}(0,1)$ and Z_t independent of σ_t.

3) The error ε_t follows an $ARCH(q)$ process that can be expressed as:

$$h_t^2=\omega+\sum_{i=1}^{q}\alpha_i\,.\,\varepsilon_{t-i}^2 \tag{12}$$

with ω being a constant, the coefficients α_i are the $ARCH$ terms assumed to satisfy $\omega>0$ and $\alpha_i\geq 0$ for $i\geq 1$ to ensure the positivity of the conditional variance h_t^2. From this definition, we can deduce the relationships:

$$\mu_t=E(X_t/\Im_{t-1})=\psi_0+\sum_{i=1}^{\bar{p}}\psi_{t-i}\,.\,X_{t-i}+\sum_{j=1}^{\bar{q}}\phi_{t-j}\,.\,\varepsilon_{t-j} \tag{13}$$

$$\sigma_t^2=Var(X_t/\Im_{t-1})=Var(\varepsilon_t/\Im_{t-1})=h_t^2 \tag{14}$$

$$X_t=\mu_t+\varepsilon_t \tag{15}$$

Definition of the GARCH$(\mathbf{p},\mathbf{q})$model: Generalized Autoregressive Conditional Heteroskedasticity

Let (X_t) be a stationary process with $E(X_t/\Im_{t-1})=\mu_t$ and $Var(X_t/\Im_{t-1})=\sigma_t^2$. We say that (X_t) is an $ARMA(\bar{p},\bar{q})$ process with $GARCH(p,q$ errors if:

1) (X_t) is an $ARMA(\bar{p},\bar{q})$ process satisfying:

$$X_t=c+\sum_{i=1}^{\bar{p}}\psi_{t-i}\,.\,X_{t-i}+\sum_{j=1}^{\bar{q}}\phi_{t-j}\,.\,\varepsilon_{t-j}+\varepsilon_t \tag{16}$$

where c is a constant, ψ_i and ϕ_j are the parameters of the $ARMA(\bar{p},\bar{q})$ model, and ε_t is the error that follows an i.i.d. process satisfying:

$$E(\varepsilon_t)=0 \qquad Var(\varepsilon_t)=\sigma^2 \qquad cov(\varepsilon_t,\varepsilon_s)=0 \text{for } t\neq s \tag{17}$$

2) The error ε_t satisfies:

$$E(\varepsilon_t/\Im_{t-1})=0 \qquad Var(\varepsilon_t/\Im_{t-1})=h_t^2 \qquad \varepsilon_t=h_t.Z_t \tag{18}$$

with $h_t>0$ and Z_t a white noise such that $Z_t \rightsquigarrow \mathcal{N}(0,1)$ and Z_t independent of σ_t.

3) The error ε_t follows a $GARCH(p,q)$ process written in the form:

$$h_t^2=\omega+\sum_{i=1}^{q}\alpha_i\,.\,\varepsilon_{t-i}^2+\sum_{j=1}^{p}\beta_j\,.\,h_{t-j}^2$$

where ω is a constant, the coefficients α_i are the $ARCH$ terms, and the coefficients β_j are the $GARCH$ terms, assumed to satisfy $\omega>0$, $\alpha_i\geq 0$, $\beta_j\geq 0$ for $i\geq 1$ and $j\geq 1$ to ensure the positivity of the conditional variance h_t^2. From this definition, we can deduce:

$$\mu_t=E(X_t/\Im_{t-1})=\psi_0+\sum_{i=1}^{\bar{p}}\psi_{t-i}\,.\,X_{t-i}+\sum_{j=1}^{\bar{q}}\phi_{t-j}\,.\,\varepsilon_{t-j} \tag{19}$$

Definition of the IGARCH$(\mathbf{p},\mathbf{d},\mathbf{q})$ Model: Integrated Generalized Autoregressive Conditional Heteroskedasticity

The $GARCH(p,q)$ process can be expressed as an $ARMA$ process for the square of the error ε_t^2:

$$\varepsilon_t^2=\omega+\sum_{k=1}^{r}(\alpha_k+\beta_k)\varepsilon_{t-i}^2+\eta_t-\sum_{j=1}^{p}\beta_j\,.\,\eta_{t-j} \tag{20}$$

with $\alpha_k=0$ if $k>q$ and $\beta_k=0$ if $k>p$ and $r=max(p,q)$:

$$\eta_t=\varepsilon_t^2-h_t^2 \tag{21}$$

Let's note that h_t^2 is the forecast of ε_t^2 based on its own lagged values:

$$Var(\varepsilon_t/\Im_{t-1}) = h_t^2 = E(\varepsilon_t^2/\Im_{t-1}) - \left(E(\varepsilon_t/\Im_{t-1})\right)^2 = E(\varepsilon_t^2/\Im_{t-1})$$

Therefore, $\eta_t = \varepsilon_t^2 - h_t^2$ is the error associated with this forecast. It can be deduced that η_t is white noise. According to equation (20), we can say that ε_t^2 is an $ARMA(r,p)$ process with the lag polynomial $\Phi(L) = 1 - \sum_{k=1}^{r}(\alpha_k + \beta_k).L^k = 1 - \sum_{i=1}^{q}\alpha_i.L^i - \sum_{j=1}^{p}\beta_j.L^j$ and the moving average polynomial $\Psi(L) = 1 - \sum_{j=1}^{p}\beta_j.L^j$, which can be written as:

$$\Phi(L) = 1 - \alpha(L) - \beta(L) \qquad \Psi(L) = 1 - \beta(L) \tag{22}$$

where

$$\alpha(L) = \sum_{i=1}^{q}\alpha_i.L^i \qquad \beta(L) = \sum_{j=1}^{p}\beta_j.L^j \tag{23}$$

The equation (20) can be written as follows:

$$\left(1 - \alpha(L) - \beta(L)\right)\varepsilon_t^2 = \omega + \left(1 - \beta(L)\right)\eta_t \tag{24}$$

The sufficient condition for the positivity of ε_t^2 is $\omega > 0$, $\alpha_k \geq 0$, and $\beta_k \geq 0$ for $1 \leq k \leq r$. The process is covariance stationary if the lag polynomial $\Theta(L)$ has all its roots outside the unit circle, which is equivalent to the condition:

$$\sum_{k=1}^{r}(\alpha_k + \beta_k) < 1$$

The lag polynomial $\Theta(L)$ of the process ε_t^2 could have a unit root, which is expressed by the condition:

$$\sum_{k=1}^{r}(\alpha_k + \beta_k) = \sum_{i=1}^{q}\alpha_i + \sum_{j=1}^{p}\beta_j = 1 \tag{25}$$

Engle and Bollerslev (1986) referred to a model satisfying (25) as an integrated $GARCH$ process, denoted $IGARCH$. The $IGARCH(p,q)$ process of integration order $d = 1$ is defined by:

$$\Phi(L)(1-L)\varepsilon_t^2 = \omega + \Psi(L)\eta_t \tag{26}$$

where the polynomials $\Phi(L)$ and $\Psi(L)$ have all their roots outside the unit circle.

Definition of the FIGARCH$(\mathbf{p,d,q})$ model (Baillie et al. BBM method): Fractionally Integrated Autoregressive Conditional Heteroscedasticity

A key characteristic of $IGARCH$ models is that the impact of past squared shocks $\eta_{t-j} = \varepsilon_{t-j}^2 - h_{t-j}^2$ for $j > 0$ on ε_t^2 is persistent. Using a parallel with $ARMA$ and $ARFIMA$ processes, Baillie et al. (1996) extended the $IGARCH$ process by allowing the integration parameter to belong to the interval $[0,1]$. They thus defined the $FIGARCH(p,d,q)$ (BBM) process:

$$\Phi(L)(1-L)^d\varepsilon_t^2 = \omega + \left(1 - \beta(L)\right)\eta_t \tag{27}$$

where the polynomials $\phi(L) = \sum_{i=1}^{q}\phi_i.L^i$ and $1 - \beta(L) = 1 - \sum_{j=1}^{q}\beta_j.L^j$ have all their roots outside the unit circle. From the (27) model, we can deduce the equation for conditional volatility h_t^2:

$$h_t^2 = \omega\left(1 - \beta(L)\right)^{-1} + \left[1 - \left(1 - \beta(L)\right)^{-1}\Phi(L)(1-L)^d\right]\varepsilon_t^2 \tag{28}$$

The model parameters can be estimated either through the Baillie et al. (1996) (BBM) approach or the Chung (1999) approach.

Definition of the FIEGARCH$(\mathbf{p,d,q})$model: Fractionally Integrated Exponential GARCH

To capture the asymmetry phenomenon, Bollerslev and Mikkelsen (1996) proposed the Fractionally Integrated Exponential $GARCH$ $(FIEGARCH)$ process described by:

$$ln(h_t^2) = \omega + \Phi(L)^{-1}(1-L)^{-d}\left(1 + \alpha(L)\right)g(z_t) \tag{29}$$

where $d \in [0,1]$ and

$$g(z_t) = \theta_1 z_t + \theta_2\left(|z_t| - E(|z_t|)\right) \qquad z_t = \varepsilon_t/h_t \qquad E(|\varepsilon_t/h_t|) = \sqrt{2/\pi} \tag{30}$$

In this model, the parameters θ_1 and θ_2 represent, respectively, the impact of the sign and the effect of the magnitude. More precisely, good news has an impact of $(\theta_1 + \theta_2)$ on volatility, while bad news has an impact of $(\theta_1 - \theta_2)$ on volatility. For $\theta_1 > 0$ and $\theta_2 > 0$, positive shocks will have a greater influence on volatility than negative shocks; for $\theta_1 < 0$ and $\theta_2 > 0$, negative shocks result in larger volatility changes than positive shocks.

Definition of the FIAPARCH$(\mathbf{p}, \mathbf{d}, \mathbf{q})$ model: Fractionally Integrated Asymmetric Power ARCH

Tse(1998) proposed the $FIAPARCH(p, d, q)$ model, which incorporates the asymmetric power $ARCH$ of Ding et al. (1993). The $FIAPARCH(p, d, q)$ model is defined by:

$$h_t^\delta = \omega + \left[1 - \left(1 - \beta(L)\right)^{-1} \Phi(L)(1 - L)^d\right] (|\varepsilon_t| - \gamma\varepsilon_t)^\delta \qquad (31)$$

where $d \in [0,1], \delta > 0$, and $-1 < \gamma < 1$.

The parameter γ represents the asymmetric component of the model, and when $\gamma > 0$, negative shocks have a greater impact on volatility than positive shocks, and vice versa. The conditional variance exhibits long-memory properties if $0 < \delta < 1$. The $FIAPARCH(p, d, q)$ model is retrieved when $\delta = 2$ and $\gamma = 0$. The parameters of the $FIAPARCH(p, d, q)$ model can be estimated using either the approach of Baillie et al. (*BBM*) or the approach of Chung.

Definition of the HYGARCH$(\mathbf{p}, \mathbf{d}, \mathbf{q})$ model: Hyperbolic GARCH

Davidson (2004) proposed a hyperbolic $GARCH$ (HYGARCH) model to overcome a limitation of the $FIGARCH$ process (infinite variance). The $HYGARCH(p, d, q)$ model is expressed as follows:

$$\Phi(L)\left((1 - \alpha) + \alpha(1 - L)^d\right)\varepsilon_t^2 = \omega + \left(1 - \beta(L)\right)\eta_t \qquad (32)$$

where $d \in [0,1]$; $\alpha \geq 0$; $\phi(L)$, $\beta(L)$, and η_t are defined as previously.

From the model (32), we can deduce the equation for conditional volatility.

$$h_t^2 = \omega + \left[1 - \left(1 - \beta(L)\right)^{-1} \Phi(L)\left((1 - \alpha) + \alpha(1 - L)^d\right)\right] \varepsilon_t^2 \qquad (33)$$

The $HYGARCH(p, d, q)$ model reduces to the $GARCH(p, q)$ model when $\alpha = 0$ ($\Leftrightarrow ln(\alpha) < 0$ or $d = 0$) and reduces to the $FIGARCH(p, d, q)$ model when $\alpha = 1$ ($\Leftrightarrow ln(\alpha) = 0$). If $d = 1$, then the $HYGARCH(p, d, q)$ model reduces either to a stationary $GARCH(p, q)$ ($\alpha < 1 \Leftrightarrow ln(\alpha) < 0$), to an $IGARCH(p, q)$ ($\alpha = 1 \Leftrightarrow ln(\alpha) = 0$), or to a $GARCH(p, q)$ ($\alpha > 1 \Leftrightarrow ln(\alpha) > 0$) with explosive conditional variances. The process is stationary if $0 < \alpha < 1$ and non-stationary if $\alpha > 1$.

Joint Models:

In joint models, (X_t) follows an $ARFIMA(\bar{p}, d_{ARFIMA}, \bar{q})$ process of orders $\bar{p}$, $d_{ARFIMA} \in \mathbb{Q}$, and $\bar{q}$such that:

$$\phi(L)(1 - L)^{d_{ARFIMA}}\, X_t = c + \psi(L)\varepsilon_t \qquad (34)$$

with $\phi(L)$ and $\psi(L)$ being lag polynomials of orders $\bar{p}$ and $\bar{q}$, respectively, with all roots outside the unit circle, and (ε_t) being white noise.

As for the error ε_t of the $ARFIMA(\bar{p}, d, \bar{q})$ model, it follows the $FIGARCH(p, d_{FIGARCH}, q)$, $FIEGARCH(p, d_{FIEGARCH}, q)$, $FIAPARCH(p, d_{FIAPARCH}, q)$, or $HYGARCH(p, d_{HYGARCH}, q)$ processes.

5. Results and discussion

Before testing the presence of long memory in the financial series of the Moroccan stock index, we will examine the hypothesis of a random walk using standard statistical tests such as the normality test, stationarity tests, autocorrelation tests of returns, and the variance ratio test.

We start by graphically representing the series of daily geometric returns of the *MASI* index (denoted as MASI-GReturns) during the period from 03/01/2002 to 11/08/2023.

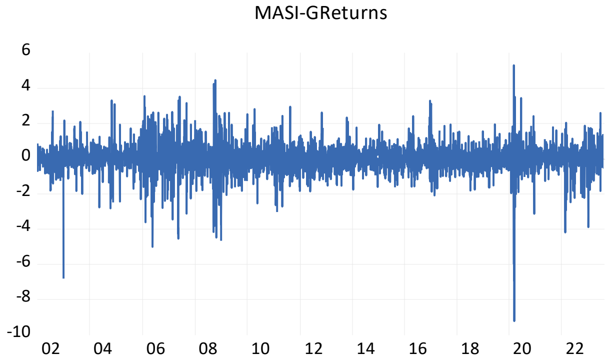

Figure 1: Daily geometric returns of the MASI Index

The figure below depicts the histogram and descriptive statistics of the geometric returns of the *MASI* index during the period from 03/01/2002 to 11/08/2023.

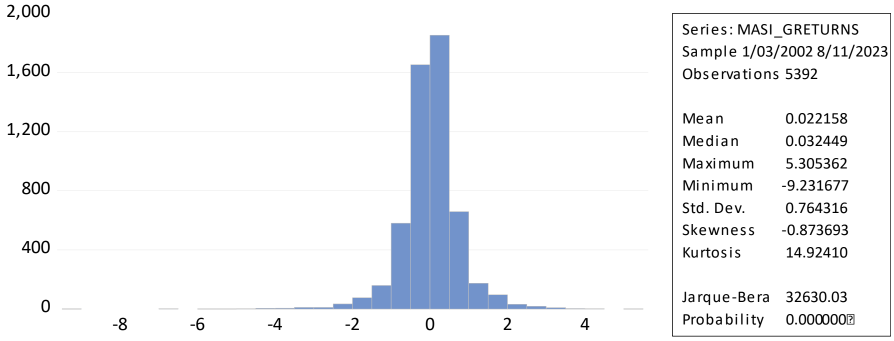

Figure 2: Histogram and descriptive statistics of geometric returns of the MASI Index

We notice that the average and median returns are positive during the study period. Returns exhibit a negatively skewed coefficient (skewness) different from 0. The kurtosis coefficient is high, exceeding three. Negative skewness and high kurtosis indicate a significant deviation from normality in the unconditional distribution of returns. The Jarque-Bera statistic rejects the hypothesis of a normal distribution of returns at a significance level of 1%.We can thus reject the null hypothesis of normality in the level series for *MASI* index returns.

The table below presents the autocorrelation and partial correlation functions of *MASI* index returns with a lag of 16.

Table 1: Autocorrelations and partial correlations of *MASI* index returns

Date: 11/07/23 Time: 13:18
Sample: 1/03/2002 8/11/2023
Included observations: 5392

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.249	0.249	333.50	0.000
		2	0.070	0.009	359.96	0.000
		3	-0.010	-0.031	360.51	0.000
		4	-0.012	-0.003	361.26	0.000
		5	0.013	0.020	362.17	0.000
		6	-0.016	-0.025	363.56	0.000
		7	0.009	0.018	364.03	0.000
		8	-0.011	-0.016	364.72	0.000
		9	0.020	0.027	366.98	0.000
		10	0.028	0.019	371.31	0.000
		11	0.013	-0.000	372.25	0.000
		12	0.031	0.027	377.34	0.000
		13	0.023	0.012	380.11	0.000
		14	0.025	0.014	383.36	0.000
		15	0.035	0.028	390.07	0.000
		16	-0.001	-0.018	390.08	0.000

The results shown in table 1 indicate that the returns of the *MASI* index exhibit significant autocorrelation throughout the study period. Indeed, the probability values corresponding to the Q-stat (LB) statistic are below the 1% significance threshold. This implies that the returns of the *MASI* index show serial dependence.

Additionally, the values of the autocorrelations oscillate around zero, indicating the absence of a long-term trend in the series, thus reflecting alternating dependence between returns.

These results lead us to reject the null hypothesis of no serial autocorrelation and accept the alternative hypothesis that the studied time series exhibits serial dependence. The result of the autocorrelation test reinforces the findings of the first non-normality test and leads us to reject the weak-form efficient market hypothesis for the Moroccan stock market.

Unit Root Test (Stationarity Test)

The standard tests used in econometrics to determine whether a process is stationary or not include the Dickey and Fuller tests (1979), the Philips-Perron test (1995), and the Kwiatkowski et al. test (1992). We applied both the *ADF* and *PP* tests to the series of geometric returns of the MASI index. The null hypothesis tested is the presence of a unit root for both the Augmented Dickey-Fuller (*ADF*) and Philips-Perron (*PP*) tests.

The results are presented in tables 2 and 3.

Table 2: Augmented Dickey-Fuller Test

Null Hypothesis: MASI_GRETURNS has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=32)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-56.95293	0.0001
Test critical values:	1% level	-3.431382	
	5% level	-2.861881	
	10% level	-2.566994	

*MacKinnon (1996) one-sided p-values.

Table 3: Philips-Perron Test

Null Hypothesis: MASI_GRETURNS has a unit root
Exogenous: Constant
Bandwidth: 2 (Newey-West automatic) using Bartlett kernel

		Adj. t-Stat	Prob.*
Phillips-Perron test statistic		-57.01014	0.0001
Test critical values:	1% level	-3.431382	
	5% level	-2.861881	
	10% level	-2.566994	

*MacKinnon (1996) one-sided p-values.

The results of the Augmented Dickey-Fuller (*ADF*) and Philips-Perron (*PP*) unit root tests indicate that the t-statistic for both tests is significantly lower than the critical values at the 1%, 5%, and 10% significance levels, suggesting that the series of geometric returns for the *MASI* index is stationary.

Variance Ratio Test

The variance ratio test was introduced by Cochrane (1988) and Lo and McKinlay (1988). It tests the hypothesis of a random walk by examining both the assumptions of homoscedasticity and heteroscedasticity. This test exploits the fact that the variance of increments in a random walk process is linear over the sampling interval. In other words, the variance of the q^{th} differences of the series is equal to q times the variance of the first differences of the series. In this test, the variance of the series data differences is compared over different intervals. The test's statistics are used to test the null hypothesis of a random walk under two different assumptions of homoscedasticity and heteroscedasticity using an asymptotic distribution.

If the variance ratio is equal to one, it means that the returns follow a random walk. To accept the null hypothesis, the joint probability should be greater than 0.05, meaning that the absolute value of the calculated z-statistic of the test must be less than the critical value of 1.96, and the variance ratio over the entire period is equal to one.

Tables 5 and 6 present the results of the variance ratio test under the two hypotheses.

Table 5: Homoscedasticity Hypothesis

Null Hypothesis: MASI_GRETURNS is a random walk
Date: 11/01/23 Time: 20:21
Sample: 1/03/2002 8/11/2023
Included observations: 5391 (after adjustments)
Standard error estimates assume no heteroskedasticity
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	27.96799	5391	0.0000
Wald (Chi-Square)	818.6302	4	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.619086	0.013620	-27.96799	0.0000
4	0.337005	0.025480	-26.02021	0.0000
8	0.168642	0.040287	-20.63569	0.0000
16	0.083697	0.059950	-15.28455	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Table 6 : Heteroscedasticity Hypothesis

Null Hypothesis: MASI_GRETURNS is a martingale
Date: 11/01/23 Time: 20:20
Sample: 1/03/2002 8/11/2023
Included observations: 5391 (after adjustments)
Heteroskedasticity robust standard error estimates
User-specified lags: 2 4 8 16

Joint Tests	Value	df	Probability
Max |z| (at period 2)*	11.28813	5391	0.0000

Individual Tests

Period	Var. Ratio	Std. Error	z-Statistic	Probability
2	0.619086	0.033745	-11.28813	0.0000
4	0.337005	0.061014	-10.86621	0.0000
8	0.168642	0.091458	-9.090012	0.0000
16	0.083697	0.122396	-7.486358	0.0000

*Probability approximation using studentized maximum modulus with parameter value 4 and infinite degrees of freedom

Under both hypotheses, the z-statistic suggests that the variance ratio is significantly different from 1 for all values of q. The results of this test indicate that the null hypothesis of a random walk is rejected for the returns of the MASI index.

- **Analysis of double long memory property**

In this section, we tested the efficiency of the Moroccan stock market by analyzing the double long memory property in conditional mean and conditional variance. We estimated four joint models, *ARFIMA-FIGARCH*, *ARFIMA-FIEGARCH*, *ARFIMA-FIAPARCH*, and *ARFIMA-HYGARCH*, under different distribution assumptions such as Normal distribution, Student's distribution, Skewed Student's distribution, and Generalized Error Distribution (*GED*).

The parameters of *ARFIMA-FIGARCH* and *ARFIMA-FIAPARCH* models were estimated using Baillie et al. (1996) (*BBM*) approach and Chung's (1999) approach.Several combinations of parameters for the four models were tested, and we selected the models with the most significant estimations. The results are presented in the tables below.

- ***Estimation Results of the joint ARFIMA-FIGARCH model***

Table 7: ARFIMA (1, d, 1)-FIGARCH (1, d, 1)Chung's method (Student distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.030323	0.011456	2.647	0.0081
d-Arfima	0.054708	0.021883	2.500	0.0124
AR(1)	0.146817	0.088729	1.655	0.0980
MA(1)	-0.051754	0.077646	-0.6665	**0.5051**
Cst(V)	0.382922	0.087703	4.366	0.0000
d-Figarch	0.290610	0.019251	15.10	0.0000
ARCH(Phi1)	-0.667964	0.18547	-3.601	0.0003
GARCH(Beta1)	-0.654827	0.19140	-3.421	0.0006
Student(DF)	5.035450	0.28241	17.83	0.0000

Table 8: ARFIMA (1, d, 1)-FIGARCH (1, d, 1) Chung's method (Skewed Student distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.020469	0.012364	1.655	0.0979
d-Arfima	0.053478	0.021868	2.446	0.0145
AR(1)	0.150827	0.088184	1.710	0.0873
MA(1)	-0.055401	0.077106	-0.7185	**0.4725**
Cst(V)	0.381939	0.089317	4.276	0.0000
d-Figarch	0.290861	0.019485	14.93	0.0000
ARCH(Phi1)	-0.654630	0.19700	-3.323	0.0009
GARCH(Beta1)	-0.641378	0.20369	-3.149	0.0016
Asymmetry	-0.034785	0.017367	-2.003	0.0452
Tail	5.044149	0.28286	17.83	0.0000

The tables 7 and 8 show that both long memory parameters *d-ARFIMA* and *d-FIGARCH* of the joint model $ARFIMA(1, d, 1)$-$FIGARCH(1, d, 1)$ (Chung's method) under the assumptions of Student's and Skewed Student's distributions are statistically significant at a 5% significance level, and all other parameters are statistically significant except for the $MA(1)$ coefficient of the first-order moving average. We conclude that the joint $ARFIMA(1, d, 1)$-$FIGARCH(1, d, 1)$ model (Chung's method) allows capturing the double long memory property in the returns and volatility of the *MASI* index.

Table 9: ARFIMA (1, d, 0)-FIGARCH (1, d, 1) Chung's method (Student distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.030521	0.011691	2.611	0.0091
d-Arfima	0.058520	0.020653	2.834	0.0046
AR(1)	0.091193	0.024865	3.668	0.0002
Cst(V)	0.375014	0.088996	4.214	0.0000
d-Figarch	0.289449	0.019274	15.02	0.0000
ARCH(Phi1)	-0.680714	0.16219	-4.197	0.0000
GARCH(Beta1)	-0.668279	0.16785	-3.982	0.0001
Student(DF)	5.043026	0.28430	17.74	0.0000

Table 10: ARFIMA (1, d, 0)-FIGARCH (1, d, 1) Chung's method (GED distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.032852	0.011773	2.791	0.0053
d-Arfima	0.055303	0.020142	2.746	0.0061
AR(1)	0.083869	0.022064	3.801	0.0001
Cst(V)	0.347716	0.082919	4.193	0.0000
d-Figarch	0.288311	0.019950	14.45	0.0000
ARCH(Phi1)	-0.651573	0.20291	-3.211	0.0013
GARCH(Beta1)	-0.639662	0.21181	-3.020	0.0025
G.E.D.(DF)	1.217042	0.034044	35.75	0.0000

Tables 9 and 10 show that both long memory parameters, *d-ARFIMA* and *d-FIGARCH*, of the joint model $ARFIMA(1, d, 0)$-$FIGARCH(1, d, 1)$ (Chung's method) under the assumptions of Student's and Generalized Error Distribution (*GED*) are statistically significant at a 1% significance level, and all other parameters are statistically significant at a 1% significance level. We conclude that the joint $ARFIMA(1, d, 0)$-$FIGARCH(1, d, 1)$ model (Chung's method) allows capturing the double long memory property in the returns and volatility of the *MASI* index.

Table 11: ARFIMA (0, d, 1)- FIGARCH (1, d, 1)
Chung's method (Student distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.031403	0.012273	2.559	0.0105
d-Arfima	0.067095	0.018282	3.670	0.0002
MA(1)	0.079285	0.019959	3.972	0.0001
Cst(V)	0.376034	0.094587	3.976	0.0001
d-Figarch	0.290031	0.019705	14.72	0.0000
ARCH(Phi1)	-0.663247	0.19928	-3.328	0.0009
GARCH(Beta1)	-0.650418	0.20451	-3.180	0.0015
Student(DF)	5.038523	0.28498	17.68	0.0000

Table 12: ARFIMA (0, d, 1)- FIGARCH (1, d, 1)
Chung's method (Skewed Student distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.022050	0.013279	1.661	0.0969
d-Arfima	0.066274	0.018150	3.652	0.0003
MA(1)	0.079404	0.019770	4.016	0.0001
Cst(V)	0.435065	0.10455	4.161	0.0000
d-Figarch	0.204518	0.033931	6.027	0.0000
ARCH(Phi1)	0.839882	0.081793	10.27	0.0000
GARCH(Beta1)	0.768215	0.10297	7.460	0.0000
Asymmetry	-0.033777	0.017465	-1.934	0.0532
Tail	5.022836	0.29490	17.03	0.0000

Tables 11 and 12 show that both long memory parameters, *d-ARFIMA* and *d-FIGARCH*, of the joint model $ARFIMA(0, d, 1)$-$FIGARCH(1, d, 1)$ (Chung's method) under the assumptions of Student's and Skewed Student's distributions are statistically significant at a 1% significance level, and all other parameters are statistically significant. We conclude that the joint $ARFIMA(0, d, 1)$-$FIGARCH(1, d, 1)$ model (Chung's method) allows capturing the double long memory property in the returns and volatility of the $MASI$ index.

Table 13: ARFIMA (0, d, 1)- FIGARCH (1, d, 1)
BBM's method (Student distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.031932	0.011984	2.665	0.0077
d-Arfima	0.066707	0.017733	3.762	0.0002
MA(1)	0.079719	0.019428	4.103	0.0000
Cst(V)	0.064450	0.010158	6.345	0.0000
d-Figarch	0.430973	0.057829	7.452	0.0000
GARCH(Beta1)	0.093650	0.056624	1.654	0.0982
Student(DF)	4.502673	0.26454	17.02	0.0000

Table 14: ARFIMA (0, d, 1)- FIGARCH (0, d, 1)
BBM's method (Student distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.031768	0.011982	2.651	0.0080
d-Arfima	0.066737	0.017757	3.758	0.0002
MA(1)	0.079845	0.019450	4.105	0.0000
Cst(V)	0.070665	0.011367	6.217	0.0000
d-Figarch	0.418470	0.046556	8.988	0.0000
ARCH(Phi1)	-0.080717	0.043579	-1.852	0.0641
Student(DF)	4.513154	0.26354	17.12	0.0000

Tables 13 and 14 show that both long memory parameters, *d-ARFIMA* and *d-FIGARCH*, of the joint models $ARFIMA(0, d, 1)$-$FIGARCH(1, d, 0)$ (BBM method) and $ARFIMA(0, d, 1)$-$FIGARCH(0, d, 1)$ (BBM method) under the assumption of the Student's distribution are statistically significant at a 1% significance level, and all other parameters are statistically significant. We conclude that the joint models $ARFIMA(0, d, 1)$-$FIGARCH(1, d, 0)$ (*BBM* method) and $ARFIMA(0, d, 1)$-$FIGARCH(0, d, 1)$ (*BBM* method) allow capturing the double long memory property in the returns and volatility of the *MASI* index.

- **Estimation Results of the joint ARFIMA-FIEGARCH model**

Table 15: ARFIMA (1, d, 0)- FIEGARCH (1, d, 0)
Normal distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.009747	0.012867	0.7575	**0.4488**
d-Arfima	0.039748	0.018833	2.111	0.0349
AR(1)	0.147680	0.025418	5.810	0.0000
Cst(V)	-0.049849	0.25424	-0.1961	**0.8446**
d-Figarch	0.434855	0.070721	6.149	0.0000
GARCH(Beta1)	0.389168	0.14803	2.629	0.0086
EGARCH(Theta1)	-0.039690	0.019283	-2.058	0.0396
EGARCH(Theta2)	0.447789	0.046820	9.564	0.0000

Table 16: ARFIMA (1, d, 0)- FIEGARCH (1, d, 0)
GED distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.029976	0.010981	2.730	0.0064
d-Arfima	0.053350	0.024599	2.169	0.0301
AR(1)	0.092696	0.030262	3.063	0.0022
Cst(V)	-1.101690	0.21195	-5.198	0.0000
d-Figarch	0.496294	0.052718	9.414	0.0000
GARCH(Beta1)	0.280907	0.11483	2.446	0.0145
EGARCH(Theta1)	-0.024510	0.016214	-1.512	**0.1307**
EGARCH(Theta2)	0.464606	0.038846	11.96	0.0000
G.E.D.(DF)	1.200035	0.034849	34.44	0.0000

Tables 15 and 16 show that both long memory parameters, *d-ARFIMA* and *d-FIEGARCH*, of the joint model $ARFIMA(1, d, 0)$-$FIEGARCH(1, d, 0)$ under the assumptions of Normal and Generalized Error Distribution (*GED*) are statistically significant at a 5% significance level, and all other parameters are statistically significant, including the constants of the normal distribution model and the *EGARCH* coefficient of the GED distribution model. We conclude that the joint $ARFIMA(1, d, 0)$-$FIEGARCH(1, d, 0)$ model allows capturing the double long memory property in the returns and volatility of the *MASI* index.

Table 17: ARFIMA(1, d, 0) -FIEGARCH(0, d, 1)
Normal distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.009750	0.0083028	1.174	**0.2403**
d-Arfima	0.040128	0.016056	2.499	0.0125
AR(1)	0.148119	0.014585	10.16	0.0000
Cst(V)	0.061798	0.26037 0	.2373	**0.8124**
d-Figarch	0.484356	0.043558	11.12	0.0000
ARCH(Phi1)	0.345742	0.14812	2.334	0.0196
EGARCH(Theta1)	-0.039306	0.020114	-1.954	0.0507
EGARCH(Theta2)	0.460323	0.047271	9.738	0.0000

Table 18: ARFIMA(1, d, 0) -FIEGARCH(0, d, 1)
GED distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.029988	0.0099449	3.015	0.0026
d-Arfima	0.053089	0.020328	2.612	0.0090
AR(1)	0.093246	0.028655	3.254	0.0011
Cst(V)	-1.128074	0.22476	-5.019	0.0000
d-Figarch	0.524007	0.037114	14.12	0.0000
ARCH(Phi1)	0.260952	0.10533	2.477	0.0133
EGARCH(Theta1)	-0.024063	0.016425	-1.465	**0.1430**
EGARCH(Theta2)	0.469530	0.037831	12.41	0.0000
G.E.D.(DF)	1.199780	0.034975	34.30	0.0000

Tables 17 and 18 show that both long memory parameters, *d-ARFIMA* and *d-FIEGARCH*, of the joint model $ARFIMA(1, d, 0)$-$FIEGARCH(0, d, 1)$ under the assumptions of Normal and Generalized Error Distribution (*GED*) are statistically significant at a 5% significance level, and all other parameters are statistically significant, including the constants of the normal distribution model and the *EGARCH* coefficient of the *GED* distribution model. We conclude that the joint $ARFIMA(1, d, 0)$-$FIEGARCH(0, d, 1)$ model allows capturing the double long memory property in the returns and volatility of the *MASI* index.

Table 19: ARFIMA(1, d, 0) -FIEGARCH(0, d, 1)
Normal distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.011518	0.015293	0.7532	**0.4514**
d-Arfima	0.069314	0.020278	3.418	0.0006
MA(1)	0.106703	0.022615	4.718	0.0000
Cst(V)	-0.215660	0.23156	-0.9313	**0.3517**
d-Figarch	0.308998	0.12957	2.385	0.0171
ARCH(Phi1)	-0.519131	0.20905	-2.483	0.0130
GARCH(Beta1)	0.830977	0.14359	5.787	0.0000
EGARCH(Theta1)	-0.043355	0.020118	-2.155	0.0312
EGARCH(Theta2)	0.458733	0.043444	10.56	0.0000

Table 20: ARFIMA(1, d, 0) -FIEGARCH(0, d, 1)
GED distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.030624	0.012756	2.401	0.0164
d-Arfima	0.060550	0.021939	2.760	0.0058
MA(1)	0.081809	0.025446	3.215	0.0013
Cst(V)	-1.091067	0.20632	-5.288	0.0000
d-Figarch	0.492003	0.052059	9.451	0.0000
GARCH(Beta1)	0.291503	0.11205	2.602	0.0093
EGARCH(Theta1)	-0.024511	0.016274	-1.506	**0.1321**
EGARCH(Theta2)	0.463862	0.039099	11.86	0.0000
G.E.D.(DF)	1.199837	0.034871	34.41	0.0000

Tables 19 and 20 show that both long memory parameters, *d-ARFIMA* and *d-FIEGARCH*, of the joint model $ARFIMA(0, d, 1)$-$FIEGARCH(1, d, 1)$ under the assumptions of Normal and Generalized Error Distribution (*GED*) are statistically significant at a 1% significance level, and all other parameters are statistically significant, including the constants of the normal distribution model and the *EGARCH* coefficient of the *GED* distribution model. We conclude that the joint $ARFIMA(0, d, 1)$-$FIEGARCH(1, d, 1)$ model allows capturing the double long memory property in the returns and volatility of the *MASI* index.

Table 21: ARFIMA(0, d, 1) -FIEGARCH(0, d, 1)
Normal distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.012106	0.014634	0.8273	**0.4081**
d-Arfima	0.066453	0.019558	3.398	0.0007
MA(1)	0.116016	0.022444	5.169	0.0000
Cst(V)	0.075103	0.26296	0.2856	**0.7752**
d-Figarch	0.485823	0.043695	11.12	0.0000
ARCH(Phi1)	0.342429	0.14848	2.306	0.0211
EGARCH(Theta1)	-0.039942	0.020403	-1.958	0.0503
EGARCH(Theta2)	0.461195	0.047471	9.715	0.0000

Table 22: ARFIMA(0, d, 1) -FIEGARCH(0, d, 1)
Student distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.030387	0.012985	2.340	0.0193
d-Arfima	0.064518	0.023528	2.742	0.0061
MA(1)	0.089337	0.034243	2.609	0.0091
Cst(V)	-0.999196	0.21282	-4.695	0.0000
d-Figarch	0.518257	0.035018	14.80	0.0000
ARCH(Phi1)	0.256648	0.094699	2.710	0.0067
EGARCH(Theta1)	-0.016128	0.016916 -	0.9534	**0.3404**
EGARCH(Theta2)	0.488159	0.036689	13.31	0.0000
Student(DF)	4.661052	0.28809	16.18	0.0000

Tables 21 and 22 show that both long memory parameters, *d-ARFIMA* and *d-FIEGARCH*, of the joint model $ARFIMA(0, d, 1)$-$FIEGARCH(0, d, 1)$ under the assumptions of Normal and Generalized Error Distribution (*GED*) are statistically significant at a 1% significance level, and all other parameters are statistically significant, including the constants of the normal distribution model and the *EGARCH* coefficient of the *GED* distribution model. We conclude that the joint $ARFIMA(0, d, 1)$-$FIEGARCH(0, d, 1)$ model allows capturing the double long memory property in the returns and volatility of the MASI index.

- **Estimation Results of the joint ARFIMA-FIAPARCH model**

Table 23: ARFIMA (1, d, 0)- FIAPARCH (1, d, 1)
Chung's Method (Normal distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.019729	0.013073	1.509	**0.1313**
d-Arfima	0.054248	0.026163	2.073	0.0382
AR(1)	0.114129	0.030815	3.704	0.0002
Cst(V)	0.355954	0.21357	1.667	0.0956
d-Figarch	0.204019	0.052417	3.892	0.0001
ARCH(Phi1)	0.849750	0.11749	7.233	0.0000
GARCH(Beta1)	0.785313	0.16151	4.862	0.0000
APARCH(Gamma1)	0.081680	0.039333	2.077	0.0379
APARCH(Delta)	2.068316	0.15963	12.96	0.0000

Table 24: ARFIMA (0, d, 1)- FIAPARCH (1, d, 1)
Chung's Method (Normal distribution)

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.019688	0.014331	1.374	**0.1696**
d-Arfima	0.070083	0.022150	3.164	0.0016
MA(1)	0.091140	0.022891	3.982	0.0001
Cst(V)	0.346985	0.21126	1.642	**0.1006**
d-Figarch	0.203927	0.052894	3.855	0.0001
ARCH(Phi1)	0.848055	0.12118	6.998	0.0000
GARCH(Beta1)	0.782802	0.16663	4.698	0.0000
APARCH(Gamma1)	0.082675	0.039309	2.103	0.0355
APARCH(Delta)	2.074937	0.15987	12.98	0.0000

Tables 23 and 24 show that both long memory parameters, *d-ARFIMA* and *d-FIGARCH*, of the joint models *ARFIMA(1,d,0)-FIAPARCH(1,d,1)* (Chung's method) and $ARFIMA(0,d,1)$-$FIAPARCH(1,d,1)$ (Chung's method) under the assumption of the *Normal* distribution are statistically significant at a 5% significance level, and all other parameters are statistically significant except for the constants of the $ARFIMA(0,d,1)$-$FIAPARCH(1,d,1)$ model. We conclude that the joint models ARFIMA(1,d,0)-FIAPARCH(1,d,1) (Chung's method) and $ARFIMA(0,d,1)$-$FIAPARCH(1,d,1)$ (Chung's method) allow capturing the double long memory property in the returns and volatility of the *MASI* index.

- **Estimation results of the joint ARFIMA-HYGARCH model**

Table 25: ARFIMA (1, d, 0)- HYGARCH (1, d, 1)
GED distribution

	Coefficient	Std.Err	t-value	t-prob
Cst(M)	0.033311	0.0091561	3.638	0.0003
d-Arfima	0.054347	0.011379	4.776	0.0000
AR(1)	0.085382	0.010966	7.786	0.0000
Cst(V)	0.063244	0.021876	2.891	0.0039
d-Figarch	0.457566	0.073335	6.239	0.0000
ARCH(Phi1)	0.087109	0.21925	0.3973	**0.6912**
GARCH(Beta1)	0.202492	0.24931	0.8122	**0.4167**
G.E.D.(DF)	1.207892	0.034592	34.92	0.0000
Log Alpha (HY)	-0.102143	0.051670	-1.977	0.0481

Table 26: ARFIMA (0, d, 1)- HYGARCH (1, d, 1)
GED distribution

	Coefficient	Std.Error	t-value	t-prob
Cst(M)	0.033580	0.012311	2.728	0.0064
d-Arfima	0.061605	0.015582	3.954	0.0001
MA(1)	0.075006	0.016351	4.587	0.0000
Cst(V)	0.063272	0.022058	2.868	0.0041
d-Figarch	0.458128	0.073454	6.237	0.0000
ARCH(Phi1)	0.088046	0.22149	0.3975	**0.6910**
GARCH(Beta1)	0.203329	0.25172	0.8078	**0.4193**
G.E.D.(DF)	1.208029	0.034583	34.93	0.0000
Log Alpha (HY)	-0.102833	0.051596	-1.993	0.0463

Tables 25 and 26 show that both long memory parameters, *d-ARFIMA* and *d-FIGARCH*, of the joint models $ARFIMA(1,d,0)$-HYGARCH(1,d,1) and $ARFIMA(0,d,1)$-$HYGARCH(1,d,1)$ under the assumption of the Generalized Error Distribution (GED) are statistically significant at a 1% significance level, and all other parameters are statistically significant except for the coefficients of the *ARCH* and *GARCH* terms. We conclude that the joint models $ARFIMA(1,d,0)$-$HYGARCH(1,d,1)$ and $ARFIMA(0,d,1)$-$HYGARCH(1,d,1)$ allow capturing the double long memory property in the returns and volatility of the *MASI* index.

In this section, we tested the efficiency of the Moroccan stock market by analyzing the double long memory property in conditional mean and conditional variance. We estimated four joint models. All the models estimated allow capturing the double long memory property in the returns and volatility of the *MASI* index, contradicting the market efficiency hypothesis.

6. Conclusion

In this study, we tested the hypothesis of weak-form informational efficiency on the Casablanca Stock Exchange during the period from 03/01/2002 to 15/08/2023. We began by testing the random walk hypothesis by applying several standard statistical tests to the series of geometric returns of the Casablanca Stock Exchange index (*MASI*), such as the normality test, stationarity tests, return autocorrelation tests, and variance ratio test.

The results obtained from these tests strongly rejected the random walk hypothesis of the Moroccan stock market over the examined period, thus concluding that the Casablanca Stock Exchange is not an efficient market in its weak form. These conclusions align with those of several studiesas mentioned above in the literature review.

We then tested the presence of double long memory in both the conditional mean and conditional variance of the geometric returns of the MASI index by applying the four joint models ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH, under different distribution assumptions such as Normal distribution, Student's distribution, Skewed Student's distribution, and Generalized Error Distribution (GED). The parameters of the *ARFIMA-FIGARCH* and *ARFIMA-FIAPARCH* models were estimated using Baillie et al. (1996) (BBM) approach and Chung's (1999) approach. Several combinations of parameters for the four models were tested, and we selected models with the most significant estimations.

The empirical results of all these models showed that both long memory parameters are statistically significant at a 1% or 5% significance level, and most other parameters are statistically significant, except for sometimes 1 or 2 parameters.

All the models estimated allow capturing the double long memory property in the returns and volatility of the *MASI* index, contradicting the market efficiency hypothesis. These results confirm that the Moroccan stock market is inefficient in its weak form.

References

[1]. Alfred M., Sivarajasingham S. (2020). Testing for Long Memory in Stock Market Returns: Evidence from Sri Lanka: A Fractional Integration Approach. SCIREA Journal of Economics. Volume 5, Issue 1.

[2]. Al-Jafari M. K., Altaee H. A. (2011). Testing the Random Walk Behavior and Efficiency of the Egyptian Equity Market. Journal of Money, Investment and Banking ISSN 1450-288X Issue 22.

[3]. Borges M. R. (2010). Efficient market hypothesis in European stock markets. The European Journal of Finance, 16:7, 711-726.

[4]. Bouchareb S., Chiadmi M. S., Ghaiti F. (2021). Long Memory Modeling: Evidence from Mediterranean Stock Indexes. WSEAS TRANSACTIONS on SYSTEMS and CONTROL. Volume 16. Pp 560- 572. DOI: 10.37394/23203.2021.16.52.

[5]. Büyükşalvarcı A., Abdioğlu H. (2011). Testing the weak form efficiency of the Turkish stock market. African journal of business management 5(34).

[6]. Chaker A. (2003). Long-Range Dependence in Daily Volatility on Tunisian Stock Market. Working Papers 0340, Economic Research Forum.

[7]. Chiny F., Mir A. (2015). Tests De l'efficience du marché financier marocain. Global journal of management and business research: C Finance. Volume 15. Issue 2. Version 1.0.

[8]. Dutta A. (2015). Efficiency tests in European equity markets. First European Academic Research Conference on Global Business, Economics, Finance and Social Sciences. Milan. Retrieved from http://globalbizresearch.org/Italy_Conference/pdf/I538.pdf.

[9]. Falloul E. (2020). Test of Weak Efficiency on Casablanca Stock Market, Chaotic Dynamic and Long Memory. Global Journal of Management and Business Research: B Economics and Commerce. Volume 20 Issue 3 Version 1.0.

[10]. Fama E. F. 1965. The Behavior of Stock-Market Prices. The Journal of Business 38 (1): 34-105.

[11]. FamaE. F. 1970. Efficient Capital Markets: A Review of Theory and Empirical Work. The Journal of Finance 25 (2): 383-417.

[12]. Fama E. F. (1991). Efficient Capital Markets. The Journal of Finance. Vol. XLVI, No. 5.

[13]. Gimba V. K. (2012). Testing the Weak-form Efficiency Market Hypothesis: Evidence from Nigerian Stock Market. CBN Journal of Applied Statistics (JAS): Vol. 3: No. 1, Article 6.

[14]. Grossman S. J., Stiglitz J. E. (1980). On the Impossibility of Informationally Efficient Markets. The American Economic Review. Vol. 70, No. 3, pp. 393-408.

[15]. Houfi M. A. (2019). Testing weak form informational efficiency on the Tunisian stock marketusing long memory models. American Journal of Financial Management. 2019, 2:6. (ISSN: 2641-4589).

[16]. Jaihan Z., Syed. A. S. (2013). An empirical study on weak market efficiency of Karachi Stock Exchange. Business Review – Volume 8 Number 1.

[17]. Kang S. H., Yoon S-M. (2006). Asymmetric Long Memory Feature in the Volatility of Asian Stock Markets. Asia-Pacific Journal of Financial Studies v35 n5 pp175-198.

[18]. Lamouchi A. R. (2020). Long Memory and Stock Market Efficiency: Case of Saudi Arabia. International Journal of Economics and Financial Issues, 2020, 10(3), 29-34.

[19]. Mahboob M. A., Tiwari A. K. and Naveed R. (2017). Impact of return on long-memory data set of volatility of Dhaka Stock Exchange market with the role of financial institutions: An empirical analysis. Banks and Bank Systems· August 2017. DOI: 10.21511/bbs.12(3).2017.04.
[20]. Maheshchandra J. P. (2014). Long Memory Volatility of Stock Markets of India and China. International Journal of Science and Research. Volume 3. Issue 7.
[21]. McKerrow A. (2013). Random Walks in Frontier Stock Markets. Ghanaian Journal of Economics, 1: 87-103.
[22]. Mollah A.S. (2007). Testing Weak-Form Market Efficiency in Emerging Market: Evidence from Botswana Stock Exchange. International Journal of Theoretical and Applied Finance, 10, 1077-1094.
[23]. Nazarian R., Naderi E., Alikhani N. G., Ashkan A. (2014).Long Memory Analysis: An Empirical Investigation. International Journal of Economics and Financial Issues Vol. 4, No. 1, pp.16-26.
[24]. Odonkor A. A., Nkrumah A. E. N., Darkwah E. A., Andoh R. (2022).Stock Returns and Long-range Dependence. Global Business Review 23(1) 37–47.
[25]. Phan K. C., Zhou J. (2014). Market efficiency in emerging stock markets: A case study of the Vietnamese stock market. Journal of Business and Management. . Volume 16, Issue 4. Ver. IV, PP 61-73.
[26]. Turkyilmaz S. et Balibey M. (2014). Long Memory Behavior in the Returns of Pakistan Stock Market: ARFIMA-FIGARCH Models. International Journal of Economics and Financial Issues. Vol. 4, No. 2, 2014, pp.400-410.
[27]. Urrutia J. L. (1995). Tests of random walk and market efficiency for Latin American emerging equity markets. Journal of Financial Research, Southern Finance Association; Southwestern Finance Association, vol. 18(3), pages 299-309.
[28]. Ziky M. et Ouali N. (2021). Mémoire longue et efficience du marché boursier : Cas du Maroc. Revue Alternatives Managériales et Economiques. Vol 3, No 2 (Avril, 2021) 202-219.