

Physics of Sounds, Music and Culture – Mathematical Foundations for Instrumentation Strategies for Teaching Acoustics

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Abstract: This article develops a theoretical model for the study of vibrating strings and resonance, which can be explored and discussed in the mutual study of Physics and Art. The results demonstrate that the equations obtained in this mathematical model can be used for activities that encompass a diversity of scientific concepts associated with everyday practices, highlighting the cultural relevance of Physics in modern society. After studying the concepts covered in the developed activities, it was observed that it is possible to work with the mathematical models applied in the interpretation of phenomena, even in scientific dissemination activities. Ultimately, the interface between art and science allows not only for the description of natural phenomena but also for the appreciation of artistic production.

Keywords: Art; Instrumentation for teaching; Instrumentation; Science dissemination; Museums.

1. Introduction

This paper presents a theoretical framework developed for scientific outreach activities that integrate the arts, aimed at fostering an effective environment for implementing instructional strategies in the teaching of Acoustics, particularly relevant for physics graduate students. Across various regions, Physics Education faces significant challenges, primarily due to insufficient laboratory resources in schools and an overemphasis on problem-solving exercises, see [1], which often detracts from meaningful conceptual discussions [2]. Consequently, many students struggle to connect classroom knowledge with real-life applications, which can render the subject matter dull and unengaging [3, 4]. Music serves as a valuable illustration of the synthesis between cultural elements and scientific understanding, engaging with disciplines such as acoustics, neuroscience, and sociobiology, while also functioning as a mode of expression and human interaction.

In response to the need for adaptive learning strategies during periods of social isolation, a range of instructional techniques were employed, including technological resources [5-8]. The objective of this study is to outline the theoretical equations that will inform the analysis of results from artistic-scientific instrumentation activities designed for both scientific outreach and physics education. These activities utilized music as a medium to elucidate acoustic phenomena and wave principles. The teaching materials developed were implemented in both remote and in-person formal education settings as supplementary resources. During this time, various educational tools aimed at teaching acoustics were created for use in physics education. Additionally, technological platforms and social networks were leveraged to engage the public and enhance their scientific literacy.

2. Theoretical background

Among the various topics addressed throughout the project, the physical concept of resonance was adopted as central, a phenomenon likely to occur in oscillatory systems subject to the action of an external periodic force. This type of force is responsible for constantly supplying energy to the oscillator, despite the dissipative effects [10]. Because of this, the oscillations produced are said to be forced. Taking these characteristics into account, Newton's 2nd law gives us Eq. (01) for the case of a harmonic force:

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = \left(\frac{F_0}{m}\right) \sin(\omega t) \quad (01)$$

In equation (1) is a non-homogeneous ordinary differential equation, x is the position, \dot{x} point is its first derivative with respect to time, \ddot{x} two points is its second derivative of time, F_0 is the magnitude of the external force, m is the mass of the oscillator, ω is the angular frequency and t is the time. The solution is given by the sum of the solution of the associated homogeneous equation with the particular solution [09]. One way to find the particular solution is to apply the complex exponential method [11,12], where Eq. (01) becomes the real part related to Eq. (02):

$$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = \left(\frac{F_0}{m}\right) e^{i\omega t} \quad (02)$$

Therefore, we propose a solution of the type

$$z(t) = Ae^{(\omega t - \varphi)} \quad (03)$$

Substituting Eq. (3) in Eq. (2), we obtain the amplitude of the forced oscillation:

$$A(\omega) = \frac{F_0}{m[(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2]^{1/2}} \quad (04)$$

Formally, resonance occurs when ω approaches ω_0 , that is, when the angular frequency of damped oscillations approaches the natural angular frequency of the system in the absence of dissipative resistance. In this situation, the amplitude given by Equation (04) reaches a peak response. This effect plays an important role in oscillations of the eardrum subjected to the action of sound waves or when a vibrating string is set to oscillate by a periodic force whose frequency coincides with the frequency of one of its normal modes [10, 11].

Another important concept addressed concerns the phenomenon of beats, which occurs when two waves of the same amplitude and very close frequencies are superimposed. Assuming two one-dimensional harmonic waves of the type

$$y(x, t) = A \cos(kx - \omega t) \quad (05)$$

With slightly different frequencies and wave numbers, the wave resulting from their superposition will be given by

$$y(x, t) = 2A \cos\left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t\right) \cos\left(\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t\right) \quad (06)$$

Where the first cosine factor corresponds to its amplitude. With this, we see that the angular frequency of the amplitude of the wave is $\Delta\omega/2$ and, as it passes through a maximum and a minimum at each cycle, the frequency of the beats is given by

$$f_{bat} = f_1 - f_2 \quad (07)$$

Being $f_1 > f_2$. This tells us that the frequency of the beats is equal to the difference between the frequencies of the two waves, and this fact is used by musicians in tuning their instruments when comparing the sound produced with a standard frequency [12]. In the case of string instruments, tuning takes place by changing the string tension by moving the tuning peg, so that the resulting sound frequency is given by Eq. (08):

$$f_n = \frac{n}{2L} \cdot \sqrt{\frac{T}{\mu}} \quad (08)$$

Where n is any integer other than zero, L is the length of the string, T is the tension to which it is subjected, and μ is its linear density. The above relationship shows that the frequency is directly proportional to the square root of the string tension.

3. Methodology and Experimental Details

Practical experiments were designed to explore normal modes of vibration in strings and resonance phenomena, utilizing a guitar as the primary instrument. The physical concepts underlying these phenomena were articulated based on the formal framework presented herein. The main aim of the study was to enhance the understanding of the relationship between sound frequency and string tension, which was exemplified through an experiment conducted with an electric guitar. Additionally, in-depth discussions were held concerning the properties and phenomena related to sound, stationary waves, and acoustics. An initiative was also undertaken to demonstrate the phenomenon of beats through hands-on activities, culminating in an experiment that employed the audiovisual editing software Audacity. The essential materials utilized in these activities included a smartphone, string, speaker, weights, and guitar.

4. Results and discussion

The speakers were connected to a computer's audio output, utilizing an audio track generated by Audacity to create vibrations at a predetermined frequency, which were then transmitted to a string. The student was tasked with evaluating the effects of frequency variation within the system, alongside analyzing equation (8) by systematically adjusting the string length, "l." This procedure was repeated for different values of suspended masses. Concurrently, a remote session was conducted via the Google Meet platform, where an experiment using an electric guitar was designed to explore the phenomenon of vibrating strings. This involved a conceptual discussion regarding the parameters outlined in Eq. (08). The relationship between the frequency and tension of the strings was assessed using smartphone applications for instrument tuning. Key properties of sound waves discussed in the forum included amplitude, frequency, speed, wave compression, and period, as well as wave phenomena such as reflection, refraction, diffraction, resonance, interference, and harmonic series [10]. Figure 1 illustrates the wave pattern produced by Audacity software, which was utilized for comparison with the wave pattern generated by the guitar. The analyses included the evaluation of parameters such as frequency and wavelength, as determined by Equations 6 and 7.

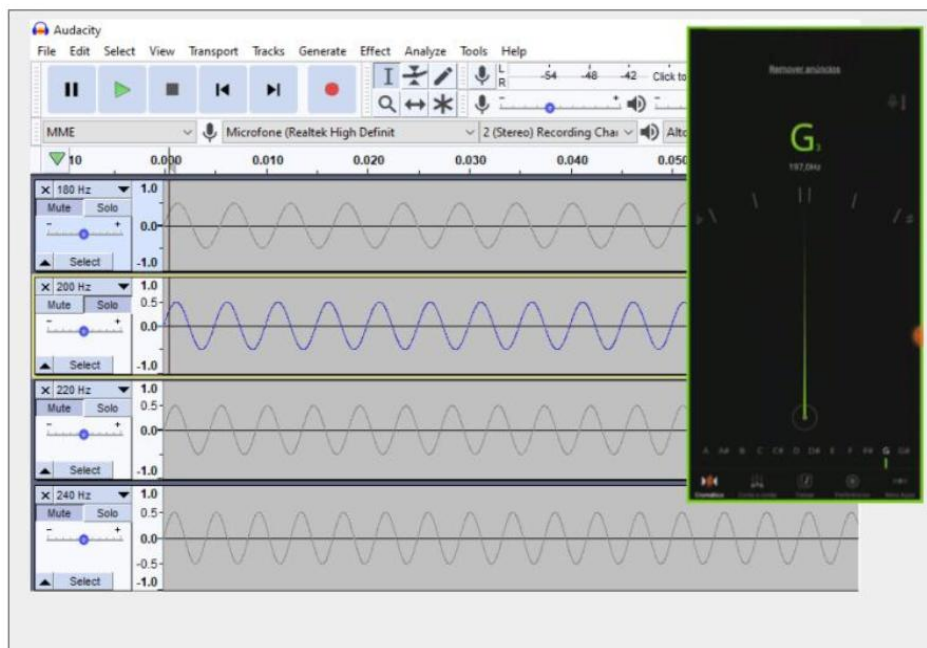
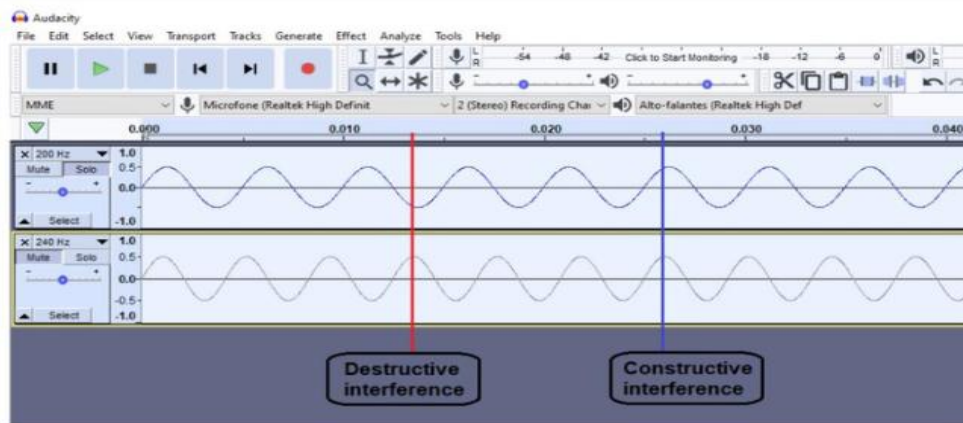
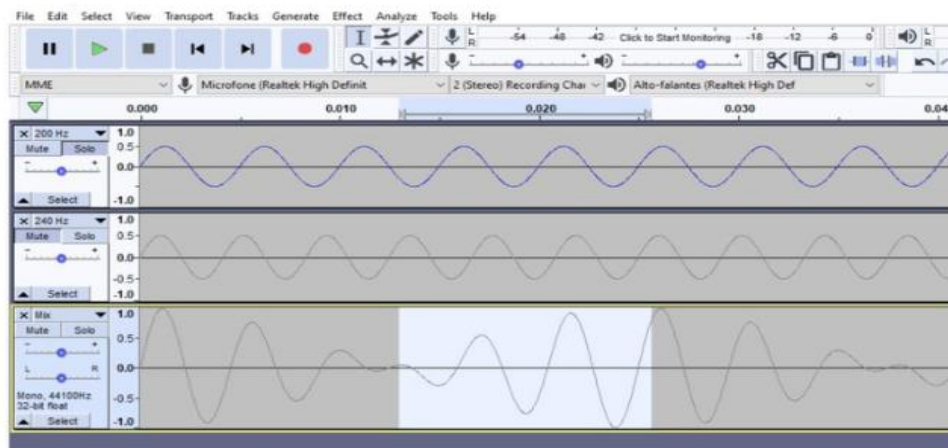


Figure 1 – Generation of tones with the Audacity software, application used and indication of the frequency of the note.

Students utilized their smartphones to tune their instruments and measure emitted frequencies. Subsequently, the procedure was repeated with a guitar, allowing for the visualization of how frequency increased as string tension was adjusted by turning the tuning machine on the guitar's head. Figure 2 presents the results from a separate activity focused on the phenomenon of beats. An experiment was conducted using Audacity to generate sine waves at slightly different frequencies (200 Hz and 240 Hz). Students were asked to evaluate the amplitude and determine how many repetitions of the beat pattern occurred over a 0.2-second interval. Their observations were documented. The waves were compared, and the phase points for constructive and destructive interference were estimated. The superposition of the initial sine waves resulted in a beat phenomenon, characterized by a sound wave with periodically varying intensity; this was graphically represented through the processing of the two initial waves into a single audio track. Additionally, the interferences corresponding to the identified phase points were observed (see Figure 2-b).



(a)



(b)

Figure 4 (a) Result of the identification of two points of maximum constructive and destructive interference of the waves generated in the Audacity software, (b) Formation of two beats by the superposition of the 200 and 240 Hz waves.

Equations 01 to 08, as outlined in the formalism, provide a mathematical and physical framework for interpreting sound-related phenomena. These concepts were explored in the acoustics class using Audacity software, which facilitated the generation of synthetic sounds and the recording of guitar audio.

5. Conclusions

This article provides a theoretical framework to enhance practical studies in acoustics, facilitating the analysis of audiovisual materials utilized in educational and scientific outreach contexts.

The research explores various scientific concepts in conjunction with everyday practices, enabling the contextualization of acoustic topics found in the natural environment. It was observed that the conceptual investigation of mathematical models relevant to the interpretation of acoustic phenomena can be applied in popular science initiatives, although a more thorough examination necessitates a robust mathematical foundation. Additionally, the use of Audacity software demonstrated significant efficacy in capturing and generating sounds, serving as a valuable resource for interpreting the equations derived from the theoretical model discussed in this article.

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