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Lottery Problem and Total Probability Formula

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Abstract: Taking the problem of drawing lots as an example, this paper presents the basic idea and form of the total probability formula step by step through the in-depth analysis of it. In this paper, the source of event complexity is discussed to explore the search process of the complete event group in depth, and then several typical scenarios and analysis methods for determining the complete event group are discussed to help readers understand and master the total probability formula.
Keywords: The problem of drawing lots; Complete event set; Total probability formula; Touch ball model.

I. TOTAL PROBABILITY FORMULA IS INTRODUCED BY DRAWING LOTS

A. Lottery problem

Question 1: There are 5 signatures, 1 of which has a word written in it, and 4 of which have no word in it. 5 people grab the signatures in turn, and ask whether the probability of each of them catching the signature with a word is the same [1].

Question 2: There are 5 signatures, 2 of which have characters in them and 3 of which have no characters in them. Five people grab the signatures one by one and ask whether the probability of each of them catching the signature with characters is the same [1].

B. Solution to problem 1

Let $B_i\left(i=1,2,3,4,5\right)$ indicate that the person i draws a signature, then

$$P\left(B_1\right)=\frac{1}{5},P\left(\overline{B_1}\right)=\frac{4}{5}.$$

Since there's only one signature, B_1 and B_2 can't happen at the same time, so there's $B_2 \subset \overline{B_1}$, $B_2=\overline{B_1}B_2$.

Applying the multiplication formula, $P\left(B_2\right)=P\left(\overline{B_1}B_2\right)=P\left(\overline{B_1}\right)P\left(B_2|\overline{B_1}\right)=\frac{1}{5}$.

By the same token, B_1, B_2, B_3 can't happen at the same time, so

$$B_3=\overline{B_1}\,\overline{B_2}B_3.$$

Applying the multiplication formula,

$$P\left(B_3\right)=P\left(\overline{B_1}\,\overline{B_2}B_3\right)=P\left(\overline{B_1}\right)P\left(B_2|\overline{B_1}\right)P\left(B_3|\overline{B_1}\,\overline{B_2}\right)=\frac{1}{5}.$$

We can calculate the probability of B_4 and B_5 in the same way.

C. Solution to problem 2

Let $B_i\left(i=1,2,3,4,5\right)$ indicate that the person i draws a signature, then

$$P\left(B_1\right)=\frac{2}{5},P\left(\overline{B_1}\right)=\frac{3}{5}.$$

When the second person catches the signed signature, we should note that B_1 and B_2 are not incompatible at this time, because it is possible to have two signed signatures at the same time.

Because $B_1 \cap \overline{B_1}=\phi$, $B_1 \cup \overline{B_1}=S$, so $B_1,\overline{B_1}$ is a complete set of events, $B_2=B_2B_1 \cup B_2\overline{B_1}$.

We can ge $$P(B_2)=P(B_2B_1\cup B_2\overline{B_1})=P(B_1)P(B_2|B_1)+P(\overline{B_1})P(B_2|\overline{B_1})=\frac{2}{5}.$$

II. COMPLETE SETS OF EVENTS AND FULL PROBABILITY FORMULAS

A. Complete sets of events

Let $B_1, B_2, \cdots, B_n$ be a set of events under the same randomized trial, satisfying the following two conditions:

$$(1) B_1 \cup B_2 \cup \cdots \cup B_n = S,$$

$$(2) B_i \cap B_j = \phi, i \neq j, i = 1,2,\cdots,n,\ j = 1,2,\cdots,n.$$

It is called a complete set of events. [2]

B. Full probability formulas

Let $B_1, B_2, \cdots, B_n$ be a complete set of events, then for any event A, there is

$$P(A)=\sum_{i=1}^{n} P(B_i)P(A|B_i).$$

This formula is called the total probability formula. There have been many studies on the learning skills of total

C. Total probability formula solution steps

The total probability formula is often used to calculate the probability of knowing cause and effect of a result caused by multiple causes. In the case that it is not easy to directly solve the probability of the concern event, we need to construct a set of complete events according to the specific situation, get the probability of each event in the complete event group and the conditional probability of the concern event under the condition that these events occur, and then use the total probability formula to get the probability of the concern event. The solution steps are as follows:

a. Calculate the probability of each event in the complete set of events,
b. Calculate the probability of the event of interest under the condition that each event occurs in the complete set of events.
c. Substitute the total probability formula to solve the probability of the event of interest.

The basic idea of the total probability formula is to decompose an unknown complex event into a number of known simple two mutually exclusive events and then solve, the most critical of which is to determine the complete event group, we will focus on it.

III. COMMON METHODS FOR DETERMINING A COMPLETE SET OF EVENTS

When it comes to the total probability formula, the complete set of events is an unavoidable problem. The determination of the complete set of events is the key to apply the total probability formula and solve the problem about the total probability formula. Once a complete set of events is determined to be wrong, the subsequent process is essentially meaningless. In many literatures, the discussion on the complete event group is not much and not specific, and most textbooks have not discussed it in depth, and the application involved is relatively monotonous, resulting in students not having a deep understanding and thorough grasp of the determination of the complete event group, and often having difficulties and doubts, which affects the learning of the total probability formula [3]. The following three steps help students master the method of determining a complete set of events.

A. Lists complex groups of mutually exclusive events

When using the total probability formula to calculate the probability of the event of interest, the first thing to do is to list the complex mutually exclusive event group in the same sample space, which is the prerequisite for determining the complete event group. A mutually exclusive event group refers to two incompatible event groups, and the intersection of any two events in the event group is an empty set. Here is an example to help you understand how to list complex groups of mutually exclusive events.

Example 1. We has two boxes. The first box contains 5 yellow balls and 3 black balls. The second box contains 4 yellow balls and 3 black balls. Now take a ball from each of the two boxes, exchange it and put it in the other box, then take a ball from the second box. Find the probability that the ball is yellow.

Analysis: There are four mutually exclusive cases. Use analysis to list them.

The First: Take A yellow ball from the first box into the second box, then take a yellow ball from the second box into the first box, recorded as B_1 ;

The Second: take A black ball from the first box into the second box, then take a black ball from the second box into the first box, recorded as B_2 ;

Third: Take a black ball from the first box into the second box, then take a yellow ball from the second box into the first box, recorded as B_3 ;

Fourth: Take a yellow ball from the first box into the second box, then take a black ball from the second box into the first box, marked as B_4 .

Let event A represent "finally take a ball from the second box as a yellow ball", easy to get

$$P(B_1)=\frac{5}{8}\times\frac{4}{7}, P(B_2)=\frac{3}{8}\times\frac{3}{7}, P(B_3)=\frac{3}{8}\times\frac{4}{7}, P(B_4)=\frac{5}{8}\times\frac{3}{7},$$

$$P(A|B_1)=\frac{4}{7}, P(A|B_2)=\frac{4}{7}, P(A|B_3)=\frac{3}{7}, P(A|B_4)=\frac{5}{7}.$$

So by the total probability formula, we get

$$P(A)=\sum_{i=1}^{4}P(B_i)P(A|B_i)=\frac{227}{392}.$$

B. The experiment is divided into two steps. Starting from the first experiment, the sample space is decomposed.

The experiment is divided into two steps, which is characterized in that one division of the sample space comes from all the results of the first step experiment, and the target event to be sought is the result of the second experiment. Since all possible causes of the target event come from the first step test, the key to solving the problem is to correctly find the complete set of events generated in the first step test. This case is suitable for a cause-and-effect scheme, that is, the occurrence of the target event is caused by a series of mutually exclusive causes from the first test, all of which form a complete set of events.

Example 2 Three aircraft (one leader, two wingmen) are flying together to a certain destination for bombing, but to reach the destination requires radio navigation, and only the leader has such equipment. Before reaching the destination, it must pass over the enemy's anti-aircraft gun positions, then the probability of any aircraft being shot down is 0.1, after reaching the goal, each aircraft will independently bomb, the probability of blowing up the target is 0.4, and the probability of the target being blown up.

The premise of the analysis of the target being blown up is that the plane reaches the destination, that is, the reason for the target being blown up is that the plane can fly to the destination, and the situation of the plane reaching the destination is divided into three situations: only the long plane reaches the destination, the long plane and a wingman reach the destination, and three planes reach the destination.

Let event A mean "target destroyed", B_1 mean "only the leader arrived at the destination", B_2 mean "the leader and a wingman arrived at the destination", B_3 mean "all three aircraft flew to the destination", so a complete event group is obtained. We can figure it out

$$P(B_1)=0.9\times 0.1\times 0.1, P(B_2)=C_2^1\times 0.9\times 0.9\times 0.1, P(B_3)=0.9^3,$$
$$P(A|B_1)=0.4, P(A|B_2)=0.4+0.4-0.4^2, P(A|B_3)=3\times 0.4-3\times 0.4^2+0.4^3.$$

Therefore,

$$P(A)=\sum_{i=1}^{3}P(B_i)P(A|B_i)\approx 0.679.$$

C. Establish a complete set of events for each test that affects the target event

The actual problem is not all directly produced by the cause, but by the cause through a series of intermediate changes before the result. At this time, the test can generally be regarded as a step or stage, and the events whose probabilities are to be sought are often in the test after multiple steps or stages, and the randomized test has a hierarchical nature. It is assumed that the random process of the test is three, that is, three-layer test. The results of the first layer test directly affect the generation of the second layer test. Therefore, various results in the first layer test are regarded as various causes leading to the target events of the second layer test, which constitute a complete group of events. Then a complete event group is formed, and the result of the third layer test is the real target event. It can be seen that this kind of test is complex, but the logical relationship of the successive tests and their constituent elements are relatively clear.

Example 3: There are 4 white balls and 6 black balls in bag A, bag B and bag C are empty bags, now take 3 balls from bag A into bag B, then take 2 balls from bag B into bag C, and finally take one ball from bag C to find the probability that the last white ball is taken out.

The process of analyzing the occurrence of events is divided into two successive stages, and each stage is divided into two successive steps. In the first stage, any 3 balls are taken from bag A into bag B, and then any 2 balls are taken from bag B into bag C. Obviously, the white balls taken from bag B are affected by the situation of taking 3 balls from bag A in front, because there are four possibilities for the number of white balls taken from bag A.

Let $B_i\ (i=0,1,2,3)$ represent the number of white balls removed from bag A, which form a complete set of events. In the second stage, take any 3 balls from bag B into bag C, and then take any ball from bag C, obviously take any ball from bag C as a white ball, affected by the previous situation of taking 3 balls from bag B, because the 2 balls taken from bag B contain a total of 3 possibilities, let $C_j\ (j=0,1,2)$ represents the number of white balls taken from bag B, which also constitutes a complete event group. On the basis of it, it directly leads to the occurrence of taking a ball from the C bag as a white ball. Because two complete sets of events are involved, this problem requires two full probability formulas, which is also the complexity of this problem.

B_0, B_1, B_2, B_3 form a complete set of events, We can calculate that

$$P(B_0)=\frac{C_4^0C_6^3}{C_{10}^3}, P(B_1)=\frac{C_4^1C_6^2}{C_{10}^3}, P(B_2)=\frac{C_4^2C_6^1}{C_{10}^3}, P(B_3)=\frac{C_4^3C_6^0}{C_{10}^3}.$$

From

$$P(C_0|B_0)=\frac{C_3^2}{C_3^2}, P(C_0|B_1)=\frac{C_2^2}{C_3^2},$$

we can get

$$P(C_0)=\sum_{i=0}^{1}P(B_i)P(C_0|B_i)=\frac{1}{3}.$$

From

$$P(C_1|B_1)=\frac{C_2^1}{C_3^2}, P(C_1|B_2)=\frac{C_2^1}{C_3^2},$$

we can get

$$P(C_1)=\sum_{i=1}^{2}P(B_i)P(C_0|B_i)=\frac{8}{15}.$$

From

$$P(C_2|B_2)=\frac{C_2^2}{C_3^2},P(C_2|B_3)=\frac{C_3^2}{C_3^2},$$

we can get

$$P(C_2)=\sum_{i=2}^{3}P(B_i)P(C_0|B_i)=\frac{2}{15}.$$

Let F mean that the last thing removed is the white ball, by

$$P(F|C_0)=0,P(F|C_1)=\frac{C_1^1}{C_2^1}=\frac{1}{2},P(F|C_2)=\frac{C_2^1}{C_2^1}=1,$$

we can get

$$P(F)=\sum_{i=0}^{2}P(C_i)P(F|C_i)=\frac{2}{5}.$$

In this example, the cause goes through a series of intermediate processes, and the final result is derived from the

IV. CONCLUSION

The total probability formula contains the concept of the combination and mutual exclusion of events, as well as addition and multiplication formulas and conditional probability formulas, which provide an effective way to calculate the probability of complex events and generally simplify the probability calculation of complex events and are widely used [9]. Total probability formula plays a very important role in solving probability problems in real life and is widely used in our daily life. A flexible grasp of total probability formula can help students broaden their horizons, improve their mathematical thinking ability and their interest in exploring the unknown world [10].

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