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Edge Notched Disc Bend Testing: Advances, Challenges, and Future Prospects

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Abstract: Existing fracture testing standards mainly focus on Mode I and Mode II fracture behaviors, with specimens being costly and complex to manufacture, while the Edge Notched Disc Bend (ENDB) specimen, due to its simple geometry and low cost, has been proven to be an effective method for assessing fracture toughness in pure modes (I, II, III) and mixed modes (I/II, I/III, I/II/III); this paper reviews the current research progress on ENDB specimens, covering their development history, basic principles, fracture toughness calculation formulas, loading modes, geometry factor calculations, and the evaluation of fracture characteristics and performance in polymers, composites, and single materials, emphasizing the flexibility of ENDB specimens in testing both pure and mixed fracture modes, summarizing the integration of ENDB experiments with various prediction criteria, models, other testing techniques, and different operational environments, identifying challenges in four areas—standardization, advanced testing techniques, testing of heterogeneous materials, and testing under long-term coupled chemical and physical environments—and providing recommendations such as the introduction of new testing technologies, optimization of formulations, improvement of prediction criteria, and integration with finite element analysis, offering references for the future development of ENDB testing and promoting the advancement of fracture mechanics research in pure and mixed modes.

Keywords: Edge Notched Disc Bend, Fracture Toughness, Experimental Methodology, Numerical Simulation, Standardization

1. Introduction

Fracture toughness is a key indicator for assessing the crack resistance of engineering materials such as concrete, rock, and asphalt mixtures in practical applications. With the increasing demand for high-performance materials in fields like construction engineering, geological engineering, and aerospace, designers are placing higher requirements on the fracture performance of engineering materials. Although traditional fracture toughness testing methods and standards, such as ISO 12135-2016[1], C1609/C1609M-12[2], and RILEM TC265-TDK[3], have been widely applied, these methods are relatively limited in testing and usually only assess the toughness for Mode I and Mode II fractures, making it difficult to comprehensively reflect the fracture behavior of materials under complex stress conditions. Developing testing techniques that can assess fracture toughness in both pure and mixed fracture modes without changing the specimen geometry and accurately reflect crack propagation has become a hot topic attracting the attention of scholars.

The Edge Notch Bending Disc (ENDB) is a fracture toughness assessment method that has emerged in the past decade, capable of evaluating the fracture toughness of engineering materials under different fracture modes. This method studies the fracture characteristics of engineering materials by introducing a prefabricated crack at the edge of a disc and applying bending loads. Compared to traditional fracture specimens, ENDB specimens offer advantages such as ease of fabrication, simple geometry, ease of drilling, lower testing costs, and straightforward data processing, which have attracted the attention of researchers. In 2011, Tutluoglu and Keles[4]proposed the Square Notched Disc Bending (SNDB) testing method, using it to study the Mode I fracture toughness of andesite and marble disc specimens, and comparing the results with those obtained from traditional fracture specimens such as Semi-Circular Bending (SCB) specimens and beam specimens. They found that SNDB specimens have a smaller crack process zone, higher bending stiffness, and less influence on the fracture toughness test results. On the one hand, SNDB specimens exhibited unique advantages over beam specimens, such as the ability to adjust the thickness of the disc specimens according to the requirements, facilitating further research on size effects by subsequent scholars. On the other hand, SNDB specimens are only suitable for testing Mode I fracture toughness and lack the ability to assess complex fracture modes. Inspired by SNDB specimens, Aliha et al. [5-7]proposed the Edge Notch Bending Disc (ENDB) and Asymmetric Edge Notch Bending Disc

(A-ENDB) testing methods, which, by altering the loading modes and adjusting the angle between the loading rod and the prefabricated crack, allow for the measurement of Mode I, II, III, and Mode I/II and Mode I/III mixed-mode fracture toughness. During use, engineering materials are subjected to various stresses from factors such as vehicles, typhoons, earthquakes, and temperature, making complex fracture loading modes, even Mode I/II/III mixed loading, more likely to occur. In order to find a simple and reliable method for evaluating Mode I/II/III mixed-mode fracture toughness, Aliha et al. [8]further proposed an improved Edge Notch Bending Disc (M-ENDB) testing method, employing an asymmetric three-point bending loading scheme and using finite element software to perform rationality analysis of T-stress, stress intensity factors, and fracture plastic zones, confirming the feasibility of comprehensively evaluating fracture toughness under Mode I, II, and III loading conditions. To date, ENDB specimens have shown unique advantages in assessing both pure mode and mixed-mode fracture performance, making them an important tool for scholars researching the fracture toughness of engineering materials.

In recent years, with the increasing demand for high-performance materials, the application of ENDB testing in different materials has gradually increased; this method is capable of revealing the fracture behavior of single materials, polymers, and composites under different fracture load conditions, and these studies provide important experimental data for designers in material selection and structural design. ENDB testing shows promising application prospects in the materials field, but there are still some challenges, including insufficient standardization of testing, a lack of advanced testing techniques to interpret mechanisms, and the need for improved predictive criteria for composite fracture modes; therefore, in-depth research into the principles, methods, and applications of ENDB testing holds significant theoretical and practical value. By summarizing and analyzing existing research findings, this can provide references for future studies and promote the further development of ENDB testing technology.

2. ENDB Testing Experimental Method

2.1 Overview of Testing Principles

The basic principle of ENDB testing is to introduce a notch at the end of a material sample in the shape of a disc or cylinder (as shown in Figure 1), and apply a three-point bending load to simulate and study the crack propagation behavior under different fracture modes. By varying the loading angle and conditions, the fracture properties of the material under pure and mixed modes can be obtained.

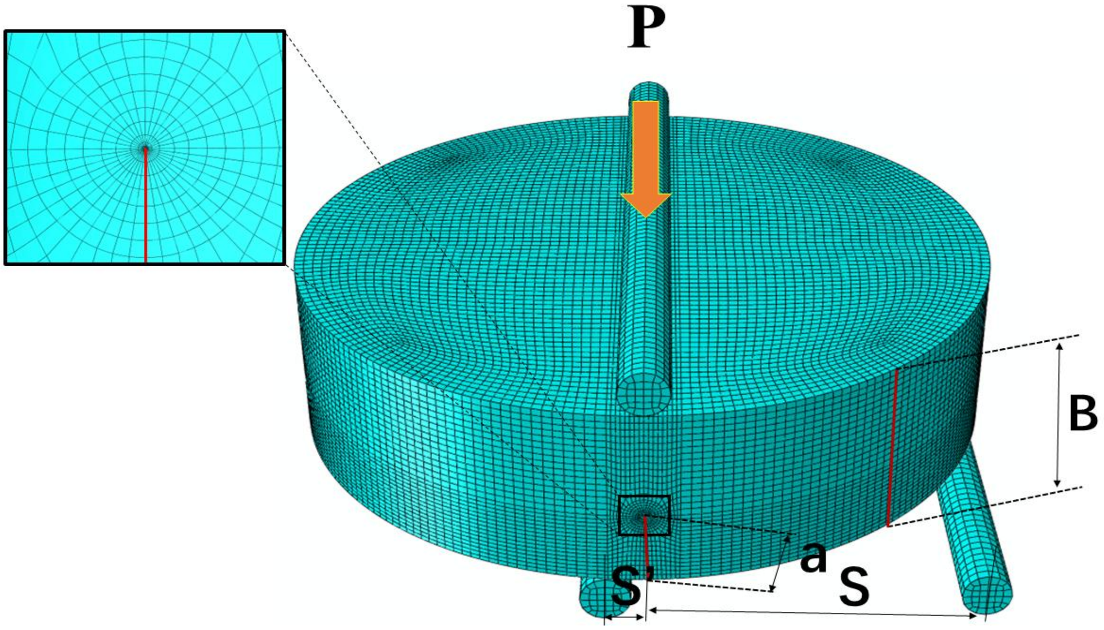


Figure 1: Schematic Diagram of the ENDB Geometry and Finite Element Mesh

2.2 Fracture Toughness and Geometric Factor Calculation Method

When the peak load P is obtained from the experiment, the formula for calculating the fracture toughnessK_i(i = Ⅰ、Ⅱ、Ⅲ)of the ENDB specimen is as follows [8]:

$$K_{IC} = Y_I\sigma\sqrt{\pi a} = Y_I\sqrt{\pi a}\frac{M}{A_M} = Y_I\sqrt{\pi a}\frac{\frac{PSS'}{(S+S')}}{\frac{2RB^2}{6}} = Y_I\sqrt{\pi a}\frac{3PSS'}{RB^2(S+S')} \tag{1}$$

$$K_{IIC} = Y_{II}\sigma\sqrt{\pi a} = Y_{II}\sqrt{\pi a}\frac{3PSS'}{RB^2(S+S')} \tag{2}$$

$$K_{IIIC} = Y_{III}\sigma\sqrt{\pi a} = Y_{III}\sqrt{\pi a}\frac{3PSS'}{RB^2(S+S')} \tag{3}$$

$$K_{eff} = \sqrt{K_{IC}^2 + K_{IIC}^2 + K_{IIIC}^2} \tag{4}$$

Where, K_{IC}、 K_{IIC} and K_{IIIC} are the fracture toughness values corresponding to fracture modes I, II, and III, respectively; K_{eff} is the effective fracture toughness, applicable for calculating the fracture toughness under mixed-mode loading; Y_I、 Y_{II} and Y_{III} aare the geometric factors, generally calculated using finite element analysis and obtained through equations (5), (6), and (7); P is the peak load obtained from the experiment; B and R are the radius and thickness of the ENDB specimen, respectively; a is the length of the pre-crack in the ENDB specimen; Sand $S'$are the distances between the bottom two supporting rollers and the center point of the disk's bottom.

$$Y_I = K_I\frac{RB^2(S+S')}{3PSS'\sqrt{\pi a}} \tag{5}$$

$$Y_{II} = K_{II}\frac{RB^2(S+S')}{3PSS'\sqrt{\pi a}} \tag{6}$$

$$Y_{III} = K_{III}\frac{RB^2(S+S')}{3PSS'\sqrt{\pi a}} \tag{7}$$

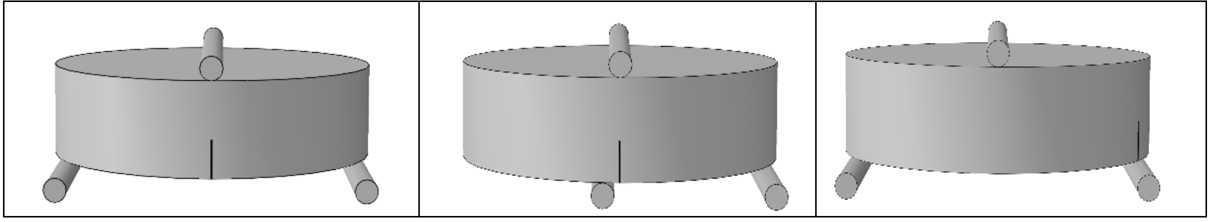
Where,K_I、 K_{II} and K_{III} are the stress intensity factors corresponding to fracture modes I, II, and III, respectively, and are generally obtained through finite element calculations.As seen from equation (5)、(6)、(7), the geometric factor is related to the specimen's crack height-to-thickness ratio, the length-to-diameter ratio, and the ratio of the distance from the loading point to the center and the radius.

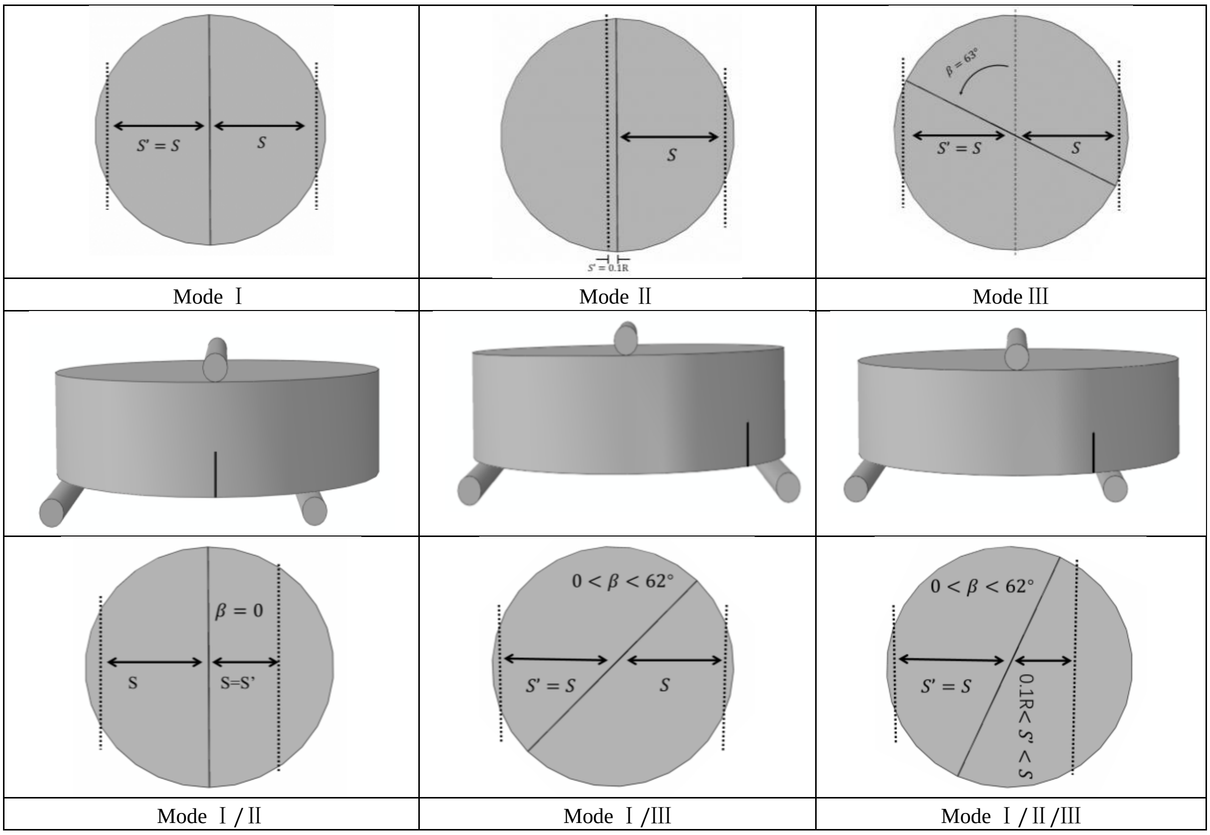
2.3 Finite Element-Based Calculation of Stress Intensity Factor

When calculating the stress intensity factor, singular elements need to be constructed at the crack tip, and both the bottom supporting rollers and the top loading rollers are modeled as rigid bodies. The elastic modulus and Poisson's ratio are assigned to the specimen in the properties. The small rods need to be assembled according to the corresponding $S$and $S'$for different fracture types. In ABAQUS, the crack faces can be separated using the "seam" feature. The overall model is shown in Figure 1.The final result needs to be taken at the crack tip at the center of the cloud disk[8].

2.4 Loading Modes and Data Processing

During the experiment, the small rods need to be placed at positions corresponding to different modes, with $S = 0.9R$typically chosen. For Mode I, III, and I/III types, $S' = S$; for Mode II, $S' = 0.1R$; for Mode I/II, $S'$is located between 0.1R and 0.9R; for Mode $I/II/III$, S' is generally chosen between 0.3R and 0.53R, with specific values set according to Figure 2.





Mode I | Mode II | Mode III

Mode I / II | Mode I /III | Mode I / II /III

Fig.2: Pure Mode I, II, III and Mixed Mode I/II, I/III, I/II/III Loading Methods and Guide Line Drawing Methods

The experiment is generally conducted using a universal testing machine for loading, and the force-displacement curve (As shown in Figure 3.) is output. The value corresponding to the highest point on the curve is taken as the value of P in equations (1), (2), and (3).

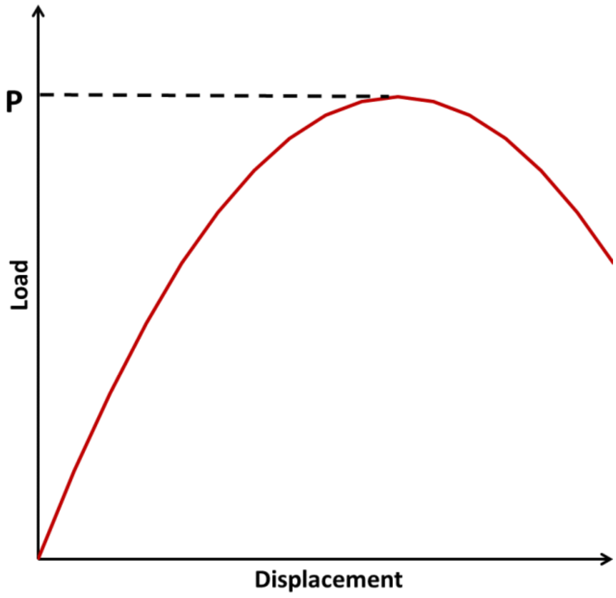


Figure 3: Schematic Diagram of ENDB Test Values

3. Applications of ENDB Testing

3.1 Applications in Polymer Materials

Polyurethane foam is a widely used material known for its light weight, excellent thermal insulation properties, and cushioning performance. Due to its unique structure, such as porosity and low density, the fracture behavior of polyurethane foam under external forces differs significantly from that of conventional materials. Understanding its fracture behavior is crucial for the optimization of its design and application. Aliha et al.[8-10] investigated the fracture performance of polyurethane foam under mixed-mode loading using ENDB specimens. Under Mode I/II loading, the crack initiation angle is typically more gradual, and cracks primarily propagate along the direction of maximum stress in the specimen, exhibiting higher fracture toughness. In contrast, under Mode I/III loading, the crack propagation path is more complex and influenced by torsional stresses, leading to a more curved crack path and a reduction in fracture toughness. The fracture toughness of polyurethane foam under mixed-mode I/II/III loading (i.e., simultaneous loading in Modes I, II, and III) is lower than its fracture toughness under pure Mode I loading. This suggests that under the combined action of multiple loading directions, the crack resistance of polyurethane foam is relatively weaker. The pores in polyurethane foam are nearly spherical, especially in closed-cell foam, where the spherical pore structure helps to disperse stress and retard crack propagation, thus enhancing the foam's crack resistance. In contrast, irregular or flattened pores can lead to localized stress concentrations, making the material more prone to crack initiation and propagation. In engineering applications, polyurethane foams with higher density and fewer pores should be used to achieve higher fracture toughness and lower crack propagation rates. Shahbazian et al.[11]conducted ENDB tests and found that although polyurethane foam exhibits high strength under compression, crack propagation remains the primary failure mode under tensile loading.

Other polymers, such as polymethyl methacrylate (PMMA), exhibit higher fracture toughness in Mode I and Mode III compared to materials like graphite, suggesting that PMMA also has resistance to crack propagation in pure crack modes as well as under anti-shear stress conditions[6]. Similar to polyurethane foam, PMMA shows higher fracture toughness under Mode I/II (a mixed mode of tension and shear). However, under Mode I + III (a mixed mode of tension and antiplane shear), the fracture behavior of PMMA involves more complex material deformation, with crack propagation paths displaying more variable characteristics[7].

3.2 Applications in composite materials

Asphalt mixtures are an essential construction material widely used in the development of transportation infrastructure, including roads, bridges, and airport runways. Composed of asphalt, mineral aggregates, fillers, fibers, and other components, asphalt mixtures are known for their strong compressive strength, durability, and ease of construction. At low temperatures, the fracture toughness of asphalt demonstrates a pronounced brittleness, particularly under Mode I loading, while Mode III exhibits relatively weaker performance[5]. Asphalt exhibits distinct brittle characteristics in its fracture performance at low temperatures, with a high crack propagation rate and poor crack resistance. At low temperatures, asphalt is prone to brittle fracture, particularly in cold regions[12]. Gu et al. [13]studied the Mode I fracture toughness of asphalt mixtures and found that, compared to the disc-shaped compact tension (DCT) and center-cracked Brazilian disc (CCBD) specimens, the ENDB specimen generates nearly zero T-stress during bending, making it more suitable for evaluating the fracture toughness of asphalt mixtures. In practical applications, increasing the thickness of the asphalt layer enhances its crack resistance. The air void content in the asphalt mixture significantly affects its fracture toughness; as the air void ratio decreases, the fracture toughness of the mixture improves. A lower air void content results in a denser mixture, thereby strengthening its crack resistance[14]. The incorporation of fibers into asphalt mixtures improves fracture resistance in both Mode I and Mode III, with a more significant effect observed in Mode III than in Mode I[15]. Compared to other commonly used fibers such as FORTA, carbon, and jute fibers, polyolefin-aramid fibers provide similar reinforcement under pure Mode I fracture conditions, but significantly enhance the fracture resistance under pure Mode II loading of asphalt mixtures[16].

Concrete is an artificial construction material composed of cement, aggregates (coarse and fine aggregates), water, fibers, and optional admixtures. It is one of the most widely used materials in modern construction and infrastructure projects, including roads, bridges, buildings, tunnels, and ports. Zhao et al.[17], using ENDB specimens, studied the fracture properties of basalt fiber-reinforced mortar and found that the fiber bridging effect of cracks led to a positive correlation between effective fracture toughness and fracture energy with roughness coefficient and fractal dimension across all loading modes. Fiber bridging helps prevent the opening of fracture surfaces and inhibits crack propagation. With a higher content of basalt fibers, the probability of fibers connecting the crack faces increases, leading to rougher fracture surfaces and greater energy consumption. Hoseini et al.[18]

noted that the increase in the volume of wave-shaped steel fibers has a positive effect on the fracture toughness of concrete with 30% and 40% coarse aggregate volume but a negative effect on concrete with 50% coarse aggregate volume. Slurry Infiltrated Fibrous Concrete (SIFCON) is a high-performance fiber-reinforced concrete known for its excellent mechanical properties and fracture toughness. In contrast to pure Mode I fracture conditions, concrete subjected primarily to Mode III (tear mode) loading exhibits a significant reduction in fracture toughness. The fracture toughness of SIFCON specimens in the mixed Mode I/III configuration is improved by up to 210.70% compared to pre-cast aggregate concrete. SIFCON also demonstrates superior fracture toughness under pure Mode III loading, with a maximum improvement of 93.89% over pre-cast aggregate concrete. The addition of fibers significantly enhances the fracture resistance of SIFCON under out-of-plane shear loading[19].Epoxy resin-filled glass tubes can effectively repair cracks and restore the fracture properties of concrete; however, as the epoxy resin content increases, the recovery rate gradually decreases. Salimi et al.[20] used epoxy resin-filled glass tubes to enhance concrete and found that the addition of 5% epoxy resin-filled glass tubes resulted in a fracture toughness recovery rate of 91.08%, but this rate declined as the epoxy resin content increased .

3.3 Application in Single Materials

ENDB, as a disc-shaped specimen, is more suitable for single materials that require sampling through drilling, such as andesite and sandstone. Gan et al.[21] found that as the contribution of Mode III increased, the fracture path of sandstone gradually deviated from the initial prefabricated crack tip, eventually exhibiting a twisted and antisymmetric plane under pure Mode III loading. Due to the complexity of the fracture surface area and the fracture path, the peak load of sandstone under pure Mode III loading is higher than that under pure Mode I loading. Aliha et al.'s[22] experimental results indicated that the T-stress in Semi-Circular Bending (SCB) specimens has a significant impact on fracture toughness, leading to higher fracture toughness in comparison to ENDB specimens. Cao et al.[23] conducted experiments using the isotropic material Chert and observed that as the angle of the bedding plane increased from 0° to 45°, the fracture toughness remained relatively constant. However, when the angle increased from 45° to 60°, the fracture toughness sharply decreased to its minimum value. Subsequently, as the angle increased from 60° to 90°, the fracture toughness rapidly recovered to a level comparable to that observed from 0° to 45°. The lowest fracture toughness was found at the 60° bedding plane angle. Liu et al.[24] discovered that the fracture toughness of limestone under Mode I is higher than that under Mode III, while in the mixed-mode configuration, the fracture toughness lies between that of Mode I and Mode III.

4. Research Progress and Challenges in ENDB Testing

4.1 Integration with Different Prediction Criteria and Models

An increasing number of prediction criteria and models have been integrated into ENDB research. Wang et al.[25]employed several criteria, including the Maximum Tangential Stress Criterion (3D-MTS), Maximum Tangential Strain Energy Density Criterion (3D-MTSED), Extended Maximum Tangential Strain Energy Density Criterion (3D-EMTSED), Maximum Tangential Strain Criterion (3D-MMTSN), Generalized Maximum Tangential Strain Criterion (3D-GMTSN), and Average Strain Energy Density Criterion (3D-MSED) to predict the fracture toughness ratios of coal-rock under different fracture modes. The 3D-MTSED and 3D-EMTSED criteria performed relatively well in three-dimensional fracture analysis, particularly when predicting the fracture toughness ratio for Mode I/III, with predicted values being in close agreement with experimental results. Zheng et al.[26] applied various three-dimensional fracture prediction criteria to forecast the fracture toughness ratio for mixed-mode I/III, finding that the improved 3D Average Strain Energy Density Criterion (3D-MSED) demonstrated strong competitiveness, successfully evaluating and predicting fracture toughness across different materials and loading conditions . Niaki et al.[27] employed deep neural network methods to predict normalized T-stress and dimensionless stress intensity factors with high accuracy and stability. He et al.[28] used both two-parameter and three-parameter Weibull statistical models to effectively analyze and predict the fracture toughness of asphalt concrete, achieving high accuracy and practical applicability.

4.2 Integration with Other Testing Techniques

Various testing techniques, such as Acoustic Emission (AE), have been applied to investigate fracture toughness. Zheng et al.[29] employed AE monitoring technology to examine the fracture toughness of shale under mixed Mode I/III loading. Analysis of the frequency distribution of AE signals revealed that under tear-dominated loading conditions, the proportion of high-frequency AE signals was significantly higher, suggesting that tearing fractures exhibit characteristics of non-planar shear deformation. Aghababaei et al.[30] utilized photogrammetry to measure the surface roughness of fracture surfaces in granite ENDB specimens. The experimental findings indicated that both fracture toughness and surface roughness exhibited a linear increase as the grain size increased. Shui et al.[31] applied Digital Image Correlation (DIC) technology to test and analyze the fracture characteristics of sandstone ENDB specimens. Through full-field strain measurement and real-time monitoring of crack propagation, they observed that the crack propagation time was advanced as the crack inclination angle increased.

4.3 Testing of ENDB under Different Operating Conditions

ENDB specimens have gradually attracted increasing attention from researchers under various operating conditions. Zarei et al.[32] investigated the fracture performance of warm-mix asphalt (WMA) mixtures subjected to freeze-thaw cycles (FTD) and found that mixtures with ENDB geometry exhibited a greater potential for crack initiation compared to those with SCB geometry, making them more susceptible to initial crack propagation after freeze-thaw damage. Gan et al.[21] studied the fracture behavior of sandstone under thermal-chemical coupling conditions, demonstrating that high temperatures and acidic environments significantly reduced the fracture toughness of sandstone, with a more pronounced decrease observed under pure Mode III loading conditions . Najjar et al.[33] examined the fracture performance of cement-emulsified asphalt mortar (CEAM) under aging conditions, revealing that aging had the most significant effect on fracture toughness under pure tensile loading, the least impact under pure shear loading, and a moderate effect under mixed-mode conditions. Zarei et al.[34] also explored the fracture characteristics of asphalt concrete under medium and low-temperature conditions. Their results showed that, at low temperatures, ENDB specimens containing 6% calcium lignosulfonate exhibited a 21% improvement in fracture toughness, while those with polyester fiber showed lower toughness. At ambient temperature, all samples exhibited a reduction in fracture toughness, suggesting that under such conditions, fracture energy methods are more appropriate for evaluation.

5. Current Challenges and Future Research Directions

5.1 Heterogeneity of Composite Materials

Asphalt concrete and other composite materials exhibit significant heterogeneity, where the crack propagation path may be influenced by aggregates, asphalt matrix, and additives, leading to uncertainty in experimental results. Future research could employ high-resolution imaging technologies, such as CT scanning, to observe crack propagation paths. This could be combined with numerical simulations to analyze the impact of material heterogeneity on fracture behavior. Additionally, optimizing the material mix (e.g., adjusting aggregate gradation, asphalt content, and additive proportions) could mitigate the effects of heterogeneity.

5.2 Geometry of ENDB Specimens

The geometry of ENDB specimens (e.g., radius, thickness, notch depth) significantly affects experimental outcomes, yet there is currently a lack of standardized guidelines. Future efforts could focus on optimizing specimen geometry through systematic experiments and numerical simulations to propose ideal size ratios. Establishing international standards for ENDB specimen geometry would ensure the comparability and repeatability of experimental results.

5.3 Complexity of Mode III Fracture

ENDB specimens are commonly used to study mixed-mode fracture behavior, but the complexity of fracture modes, particularly Mode III, complicates both experimental and theoretical analysis. Mode III fracture involves torsion and tearing, with fracture paths typically non-planar and difficult to predict accurately. The fracture paths are often characterized by anti-symmetric twisting, and traditional fracture criteria may not provide accurate predictions of fracture toughness (e.g., the 3D-MTS criterion tends to overestimate toughness when Mode III predominates). Future research could propose improved fracture criteria, such as an enhanced 3D mean strain energy density criterion (3D-MSED), which distinguishes between volumetric strain energy density and distortion strain energy density. This could improve the predictive accuracy for fracture toughness under mixed-mode I/III conditions.

5.4 Complex Multiphysical Coupling in Real-World Engineering

In practical engineering applications, materials such as rocks and concrete often experience fractures under complex multiphysical coupling environments (e.g., thermal, chemical, and mechanical stresses). However, ENDB specimens have limitations in simulating fracture behavior under such conditions. Future studies could focus on multiphysical coupling experiments that combine thermal, chemical, and mechanical stress factors to simulate conditions closer to those encountered in real-world engineering. For example, thermochemical coupling experiments (temperature + chemical corrosion), as mentioned in the literature, offer valuable insights into fracture behavior under complex environmental conditions. Further, integrating finite element analysis (e.g., ABAQUS) with experimental data could help simulate fracture behavior under complex loading conditions, thus validating the reliability of experimental results.

REFERENCES

[1]. Metallic materials - Unified method of test for the determination of quasistatic fracture toughness (Second Edition). 2016. p. 106.

[2]. Standard Test Method for Flexural Performance of Fiber-Reinforced Concrete (Using Beam With Third-Point Loading). 2013.

[3]. Xu S, Li Q, Wu Y, et al. RILEM Standard: testing methods for determination of the double-K criterion for crack propagation in concrete using wedge-splitting tests and three-point bending beam tests, recommendation of RILEM TC265-TDK %J Materials and Structures. 2021;Vol.54(No.6):1-11.

[4]. Tutluoglu L, Keles C. Mode I fracture toughness determination with straight notched disk bending method. International Journal of Rock Mechanics and Mining Sciences. 2011;48(8):1248-1261.

[5]. Aliha MRM, Bahmani A, Akhondi S. Numerical analysis of a new mixed mode I/III fracture test specimen. Engineering Fracture Mechanics. 2015;134:95-110.

[6]. Aliha MRM, Bahmani A, Akhondi S. Determination of mode III fracture toughness for different materials using a new designed test configuration. Materials & Design. 2015;86:863-871.

[7]. Bahmani A, Aliha MRM, Sarbijan MJ, Mousavi SS. An extended edge-notched disc bend (ENDB) specimen for mixed-mode I plus II fracture assessments. International Journal of Solids and Structures. 2020;193:239-250.

[8]. Aliha MRM, Kouchaki HG, Haghighatpour PJ. Designing a simple and suitable laboratory test specimen for investigating the general mixed mode I/II/III fracture problem. Materials & Design. 2023;236:24.

[9]. Aliha MRM, Linul E, Bahmani A, Marsavina L. Experimental and theoretical fracture toughness investigation of PUR foams under mixed mode I plus III loading. Polymer Testing. 2018;67:75-83. doi:10.1016/j.polymertesting.2018.02.015

[10]. Aliha MRM, Mousavi SS, Bahmani A, Linul E, Marsavina L. Crack initiation angles and propagation paths in polyurethane foams under mixed modes I/II and I/III loading. Theoretical and Applied Fracture Mechanics. 2019;101:152-161. doi:10.1016/j.tafmec.2019.02.016

[11]. Shahbazian B, Mirsayar MM, Aliha MRM, Darvish MG, Asadi MM, Haghighatpour PJ. Experimental and theoretical investigation of mixed-mode I/II and I/III fracture behavior of PUR foams using a novel strain-based criterion. International Journal of Solids and Structures. 2022;258:8. doi:10.1016/j.ijsolstr.2022.111996

[12]. Aalinejadian H, Aliha MRM, Ameri M, Karimi HR, Haghighatpour PJ, Choupani N. Comparison of mixed mode I/III fracture toughness data obtained from the ENDB specimens manufactured by gyratory cylinders with two diameters of 100 and 150 mm. International Journal of Pavement Engineering. 2023;24(2):12.

[13]. Gu XY, Cheng T, Zhang FJ, Aliha MRM. Experimental and theoretical framework for illustrating the dependency of fracture toughness with tensile, bending, and compression loading in asphaltic samples. Fatigue & Fracture of Engineering Materials & Structures. 2023;46(9):3321-3341.

[14]. Pour PJH, Aliha MRM, Keymanesh MR. Evaluating mode I fracture resistance in asphalt mixtures using edge notched disc bend ENDB specimen with different geometrical and environmental conditions. Engineering Fracture Mechanics. 2018;190:245-258.

[15]. Motamedi H, Fazaeli H, Aliha MRM, Amiri HR. Evaluation of temperature and loading rate effect on fracture toughness of fiber reinforced asphalt mixture using edge notched disc bend (ENDB) specimen. Construction and Building Materials. 2020;234:10.

[16]. Hajiloo HR, Karimi HR, Aliha MRM, et al. Crack resistance of fiber-reinforced asphalt mixtures: Effect of test specimen and test condition. Fatigue & Fracture of Engineering Materials & Structures. 2022;45(3):921-937. doi:10.1111/ffe.13647

[17]. Zhao Y, Tang W, Zhang YF, et al. Evaluating fracture resistance of basalt fiber reinforced mortar to mode I/III load using edge notched disc bend (ENDB) specimen: Insights from acoustic emission and morphological analysis. Construction and Building Materials. 2024;411:23.

[18]. Hoseini SO, Sohrabi MR, Mousavi SR, Ghasemi M. Effects of coarse aggregate and wavy steel fiber volumes on the critical stress intensity factors of modes I and III cracks in self-compacting concrete using ENDB specimens. Theoretical and Applied Fracture Mechanics. 2022;121:15.

[19]. Murali G, Abid SR, Al-Lami K, Vatin NI, Dixit S, Fediuk R. Pure and mixed-mode (I/III) fracture toughness of preplaced aggregate fibrous concrete and slurry infiltrated fibre concrete and hybrid combination comprising nano carbon tubes. Construction and Building Materials. 2023;362:16.

[20]. Salimi A, Kamboozia N, Aliha MRM. Experimental Investigation of the self-healing capability of Roller-Compacted Concrete (RCC) containing epoxy-filled glass tubes through fracture properties. Case Studies in Construction Materials. 2024;21:27.

[21]. Gan ZQ, Hua W, Wang AB, et al. Experimental investigation on mixed mode I-III fracture behaviors and degradation mechanism of sandstone under thermal-chemical stimulation. Theoretical and Applied Fracture Mechanics. 2024;129:18.

[22]. Aliha MRM, Kosarneshan K, Salehi SM, Haghighatpour PJ, Mousavi A. On the Statistical Prediction of <i>K</i>_{Ic} and <i>G</i>_{Ic} for Railway Andesite Ballast Rock Using Different Three-Point Bend Disc Samples. Rock Mechanics and Rock Engineering. 2023;56(7):5181-5202.

[23]. Cao RH, Yao RB, Dai H, Qiu XY, Lin H, Li KH. Fracture behaviour of transversely isotropic rocks under pure mode III fracture: Experiment and numerical simulation. Theoretical and Applied Fracture Mechanics. 2024;129:19. doi:10.1016/j.tafmec.2023.104208

[24]. Liu ZL, Ma CD, Wei XA. Study on fracture behavior of layered limestone under mixed mode I/III loading. Theoretical and Applied Fracture Mechanics. 2023;128:15. doi:10.1016/j.tafmec.2023.104102

[25]. Wang W, Wang CL, Zhao Y, Zheng K, Bi J. Study on I/III mixed fracture characteristics of coal in different regions of China. Theoretical and Applied Fracture Mechanics. 2024;134:12.

[26]. Zheng K, Wang CL, Zhao Y, Bi J, Liu HF. A modified 3D mean strain energy density criterion for predicting shale mixed-mode I/III fracture toughness. Journal of Rock Mechanics and Geotechnical Engineering. 2024;16(7):2411-2428.

[27]. Niaki MH, Pashaian M. Using deep learning method to predict dimensionless values of stress intensity factors and <i>T</i>-stress of edge notch disk bend (ENDB) specimen. Fatigue & Fracture of Engineering Materials & Structures. 2024;47(8):2789-2802.

[28]. He JX, Liu L, Yang HH, Aliha MRM. Using two and three-parameter Weibull statistical model for predicting the loading rate effect on low-temperature fracture toughness of asphalt concrete with the ENDB specimen. Theoretical and Applied Fracture Mechanics. 2022;121:13.

[29]. Zheng K, Wang CL, Zhao Y, Bi J, Liu HF. Theoretical and experimental exploration on the combined mode I plus III fracture toughness of shale using the edge-notched disk bending method with acoustic emission monitoring. Theoretical and Applied Fracture Mechanics. 2023;125:13.

[30]. Aghababaei M, Behnia M, Aliha MRM. Experimental investigation on the effect of grain size of granitic rocks on the fracture roughness and toughness. Geomechanics for Energy and the Environment. 2024;38:13.

[31]. Shui X, Zhou L, Zhu ZM, Nie F, Wang HH, Zhang HD. Investigation of the fracture characteristics of mixed-mode I/III crack by using two kinds of sandstone specimens. Engineering Fracture Mechanics. 2024;310:19.

[32]. Zarei M, Tabasi E, Ghandehari M, Rezaie M, Khordehbinan MW, Al-Bahrani M. Effect of hospital waste pyrolysis hydrocarbon (HWPHC) on fracture behavior of Warm Mix asphalt (WMA) under freeze-thaw damage (FTD). Construction and Building Materials. 2022;359:17.

[33]. Najjar S, Moghaddam AM, Sahaf A, Aliha MRM. Aging effect on the mixed-mode (I/III) fracture toughness of cement emulsified asphalt composite: Experimental and statistical investigation. Engineering Fracture Mechanics. 2022;264:26.

[34]. Zarei M, Kordani AA, Zahedi M. Evaluating the fracture behaviour of modified asphalt concrete composites (ACC) at low and intermediate temperatures using edge notched disc bend (ENDB) specimen. Road Materials and Pavement Design. 2022;23(8):1917-1941. doi:10.1080/14680629.2021.1950819