

Context-Dependent Data Envelopment Analysis and an Application

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Abstract: Data envelopment analysis (DEA) is a method for identifying the efficient frontier of decision making units (DMUs) which has multiple outputs and multiple inputs. DEA forms a unique efficient frontier by DMUs and it measures the DMUs' relative efficiency respect to the this frontier. Context-dependent DEA is an approach which a set of DMUs are evaluated against a particular evaluation context. In context-dependent DEA, different efficient frontiers are formed by DMUs, each of efficient frontiers are used for relative attractiveness and progress measurements as evaluation context. In this study, we used the Social Insurance Institution's Provincial Directorates database for the year 2012. We explained that a context-dependent DEA which measures relative attractiveness and progress of provincial directorates on a specific performance level against exhibiting poorer and better performance are chosen as the evaluation context, respectively. In this study number of employees, number of establishments, province population as input variables; number of insured and retired are taken as output variables. Input oriented context-dependent DEA model is used for relative attractiveness and progress measurements. Additionally, the reference sets are formed to the provincial directorates located in a specific efficiency level. Thus, regarding of the closest benchmark targets respect to their performance provincial directorates take an opportunity to the improve their performance step by step.

Keywords: Data Envelopment Analysis (DEA), Attractiveness, Progress, Context-Dependent DEA, Benchmarking

1. Introduction

In the world where globalization takes place further and further, conditions of competition among organizations has been increased. In this process, units serving in the product and service sector need to improve their performances in order to survive.

Managers who take this phenomenon into account can plan their forward looking activities in a more efficient way by analyzing the process which they are in. In order to adapt the increasing conditions of competition in recent years and catch up with current situation, both private and public institutions perform efficiency and productiveness analyses and form strategy development units.

There are two, parametric and non-parametric, main approaches to estimate efficiency limit in economy.

Parametric approach requires a functional form of dependent variable related to independent variable to be applied (like regression model or production function) and this functional form needs some assumptions about error term to be met (Luptacik, 2010).

Non-parametric approach does not use an analytic function. Non-parametric methods measure the distance to efficiency limit using mathematical programming techniques (constrained optimization).

Data envelopment analysis is a nonparametric method which does not require an assumption about the functional form. Therefore, it is more flexible compared to other methods. Also, DEA produces only one efficiency score even when multiple inputs and multiple outputs are present (Cook et al., 2014). Charnes, Cooper and Rhodes (CCR) (1978) developed the data envelopment analysis which is a mathematical programming method.

While DEA is measuring DMU relative efficiencies, it divides them into two groups as efficient and inefficient. It forms a reference set of efficient DMU's that guides inefficient DMU's to be efficient. However it is hard for inefficient DMU's to select the suitable DMU for themselves.

In this situation, context-dependent DEA, which allows inefficient DMU's to improve their performances step by step, is used.

Context-dependent DEA is an extension of DEA developed by Zhu (2003) and gives relative attractiveness and progress measures of a specific DMU.

In order to obtain relative attractiveness and progress measure in context-dependent DEA, original DEA methodology is modified where context performance is determined with regard to a special evaluation context (Lu and Hung, 2008). Evaluation contexts in context-dependent DEA represent the efficiency limit in different levels. Each efficiency limit of DMU's enables to determine the context performance of a specific DMU.

Thus context-dependent DEA allows to characterize future performances of DMU's using a combination of attractiveness and progress results (Lu and Hung, 2008).

The aim of this article is to explain the DEA and a new approach of traditional DEA, context based DEA, which are used in efficiency measurement, in detail and to examine using an application data. In this regard, efficiencies are measured using context-dependent DEA method on the input and output data belonging to 81 cities taken from one of the important institutions of Turkey, Social Insurance Institution (SII), and relative attractiveness and progress measures are obtained.

2. Material and Method

Data envelopment analysis is a method used to identify the efficiency limits of decision making units. Efficiency limit is characterized by DMU's using only one efficiency score. DEA measures relative efficiencies of DMU's with multiple inputs and outputs. Performance of DMU's depends on described the efficiency limit (Chen et al., 2005). Adding or removing an inefficient DMU does not change the efficiency limit or efficiencies of current DMU's. Inefficiency scores changes only when efficiency limit changes (Morita and Zhu, 2007). In case of performances of inefficient DMU's get worse or better, efficiency scores of efficient DMU's continue to exist. In other words, even if performances of inefficient DMU's depend on efficient DMU's, after DEA is defined, performances of efficient DMU's do not depend on inefficient DMU's (Chen et al., 2005).

However, evaluations usually affected by context. Performance of a DMU looks more attractive compared to less attractive alternatives and less attractive compared to more attractive alternatives. According to theory of consumer choice, consumer choice is affected by context as a circle looks bigger when surrounded by smaller circles and looks smaller when surrounded by bigger circles (Tversky and Simonson, 1993).

If we think about this effect in DEA's structure, context based DEA, which measures relative attractiveness of a specific DMU compared to other methods, appear. Relative attractiveness of DMU_x compared to DMU_y , depends on the presence or absence of a third choice, DMU_z (or a group of DMU s) (Chen et al., 2005). Relative attractiveness depends on an evaluation context formed from alternative choices (DMU's) (Morita and Zhu, 2007).

When context performance is determined according to a specific efficient context (evaluation context), the original DEA method is changed in order to obtain attractiveness in context-dependent DEA (Chen et al., 2005). Evaluation contexts in context-dependent DEA are obtained by dividing DMU's by efficiency limits in different levels. Each efficiency limit presents an evaluation context in order to measure relative attractiveness. If the first level (actual) efficiency limit is removed, the remaining (inefficient) DMU's form a new efficiency limit in the second level. If this new second level efficiency limit is removed, a new efficiency limit will be formed in the third level and this situation will repeat until no remaining DMU's exist. Thus each efficiency limit provides an evaluation context of relative attractiveness just like the second level efficiency limit which functions as an evaluation context to measure relative attractiveness of DMU's in the first level efficiency limit. On the other hand, it is possible to measure performance of DMU's according to the first and the second level efficiency limit (Morita and Zhu, 2007). The presence or absence of the second level efficiency limit is seem to affect DMU's in the first level efficiency limit (Chen et al., 2005).

Each efficiency limit obtained from context-dependent DEA provides an evaluation context also to measure the progress besides relative attractiveness. If it is seen that DMU's in a specific level performs equally, attractiveness measurement allows to differentiate the same performance with regard to the same evaluation context. It is possible to characterize the performance of DMU's even more by using attractiveness and progress measurements together (Khezrimotlagh and Chen, 2018).

Context-dependent DEA is an extension of the original DEA approach. The original DEA approach evaluates each DMU against a set of DMU's, but it cannot determine which efficient DMU is a better choice for the inefficient DMU. The reason for this is that all efficient DMU's have 1 efficiency score (Morita and Zhu, 2007).

2.1 Stratification of DEA Model

Let us assume that n DMU's produce s outputs using m inputs. Let J^1 be the series of all DMU's and E^1 be the series of DMU's efficient in J^1 . In that case, J^l and E^l is defined mutually as $J^{l+1} = J^l - E^l$. E^l can be found through below linear programming problem using DMU's with $\theta_k^l = 1$ optimal value.

$$\begin{aligned} \min \quad & \theta_k^l = \theta \\ \text{constraints} \quad & \sum_{j \in J^l} \lambda_j x_{ij} \leq \theta x_{ik}, \quad i = (1, \dots, m) \\ & \sum_{j \in J^l} \lambda_j y_{rj} \geq y_{rk}, \quad r = (1, \dots, s) \\ & \lambda_j \geq 0, \quad j \in J^l(1) \end{aligned}$$

In this model, x_{ij} is the i -th input of DMU_j and y_{rj} is the r -th output of DMU_j . When $l = 1$, model (1) becomes input oriented CCR model.

E^l includes all radial efficient DMU's. DMU's in E^1 defines the first level efficiency limit. When $l = 2$, model (1) gives the second level efficiency limit after the first level efficient DMU's are removed. This way, we define a couple of levels of efficiency limits. We define E^l as efficiency limit of l -th level. The below algorithm show how to find these efficiency limits using model (1) (Morita and Zhu, 2007).

Step 1: Evaluate all DMU series (J^1) using model (1) to obtain E^1 , the first level efficient DMU series, by taking $l = 1$.

Step 2: Remove efficient DMU's for future DEA applications using $J^{l+1} = J^l - E^l$. Stop if $J^{l+1} = \emptyset$.

Step 3: Evaluate J^{l+1} , a sub series of the new inefficient DMU's, to obtain the new efficient DMU series, E^{l+1} (new efficiency limit).

Step 4: $l = l + 1$. Go back to Step 2.

Stopping rule: Stop if $J^{l+1} = \emptyset$.

Model (1) stratifies all DMU series divided into different sub groups of efficiency levels. This DMU series have the following features.

$$E^l \cap E^{l'} = \emptyset \text{ for } J^1 = \cup E^l \text{ and } l \neq l'$$

if $l' > l$, then DMU's in $E^{l'}$ are suppressed by DMU's in E^l .

Each DMU in E^l series is efficient with respect to DMU's in $J^{l'}$ series for all $l' > l$.

2.2 Input Oriented Context-Dependent DEA Model

Let us consider a specific DMU_q in the level of E^{l_0} where $l_0 \in \{1, \dots, L - 1\}$. The model below is used to characterize the attractiveness measure of DMU's in E^{l_0} with regard to E^{l_0+d} , $d = (1, \dots, L - l_0)$ levels which performs worse (Chen et al., 2005).

$$\begin{aligned} \min \quad & \theta_q^d = \theta \quad \quad \quad d = (1, \dots, L - l_0) \\ \text{constraints} \quad & \sum_{j \in E^{l_0+d}} \lambda_j x_{ij} \leq \theta x_{iq} \quad \quad i = (1, \dots, m) \\ & \sum_{j \in E^{l_0+d}} \lambda_j y_{rj} \geq y_{rq} \quad \quad r = (1, \dots, s) \\ & \lambda_j \geq 0, \quad j \in E^{l_0+d}(2) \end{aligned}$$

For each $d = 1, \dots, L - l_0$, we can say $\theta_q^{d_1} > \theta_q^{d_2}$ where $\theta_q^{d_1} > 1$ and $d_1 > d_2$. In that case, θ_q^d is called input oriented d -degree attractiveness of DMU_q of a specific E^{l_0} level. Each efficiency level in the model (2) represents an evaluation context for relative attractiveness of DMU's in E^{l_0} . The larger the $\theta_q^d > 1$ value, more attractive the DMU_q will be. Because DMU_q makes itself more unique (distinctive) than evaluation context E^{l_0+d} . We can sort DMU's in E^{l_0} series based on their attractiveness scores and determine the best one.

We use the following model in order to obtain the progress measure related to E^{l_0-g} , $g = (1, \dots, l_0 - 1)$ levels, which performs better, of a specific DMU_q in the E^{l_0} series where $l_0 \in \{2, \dots, L\}$ (Lu and Hung, 2008).

$$\begin{aligned} \min \quad & \varphi_q^g = \varphi \quad \quad \quad g = (1, \dots, l_0 - 1) \\ \text{constraints} \quad & \sum_{j \in E^{l_0-g}} \lambda_j x_{ij} \leq \varphi x_{iq} \quad \quad i = (1, \dots, m) \\ & \sum_{j \in E^{l_0-g}} \lambda_j y_{rj} \geq y_{rq} \quad \quad r = (1, \dots, s) \\ & \lambda_j \geq 0, \quad j \in E^{l_0-g}(3) \end{aligned}$$

We can say $\varphi_q^{g_1} < \varphi_q^{g_2}$ for $\varphi_q^g < 1$ and $g_1 > g_2$ where $g = 1, \dots, l_0 - 1$. In that case, φ_q^g is called as input oriented g -degree progress of DMU_q in the level of a specific E^{l_0} . The smaller the $\varphi_q^g < 1$ value, the higher the progress of DMU_q will be.

2.3 Reference Sets

In order to improve the performances of inefficient DMU's, the improvement target need to be given among efficient DMU's. Reference set suggests an improvement target for inefficient DMU's. Actually, when $l = 1$, model (1) gives reference set among efficient DMU's for inefficient DMU's. This may be the final improvement target. However this aim for some of the inefficient DMU's can be completely different from the current performance and improvement can be difficult to provide (Morita and Zhu, 2007).

λ density vector in Model (2) represents the $R_k^{CCR}(l)$ reference set of KVB_k in the l_k efficiency level based on evaluation context, E^l , where $l < l_k$.

$$R_k^{CCR}(l) = \{j \in J^l | \lambda_j > 0\} \quad (4)$$

3. Application

Together with globalization and technological advances, there is a rapid change and transformation in many areas. Private and public institutions carry out their activities within this scope in order to adapt these advancements and response to expectations in a better way.

SII; provides social security services by securing people in terms of social insurance and general health insurance.

In this article, context-dependent DEA method is applied on SII provincial directorates based on this course of action and progress measures are obtained.

Firstly, provincial directorates are divided by their efficiency limits in different levels. Each obtained efficiency limit is used as an evaluation context in a specific performance level.

Then, provincial directorates, which performed worse or better, are selected as evaluation context respectively using context-dependent DEA and attractiveness and progress measures are obtained.

Also, a reference set is formed for provincial directorates with low performances from provincial directorates with different efficiency level and better performances. This way, possibility to performances of provincial directorates with low performances step by step is enabled and accessible comparison targets are provided. In the application, WINQSB packet program is used and results are obtained. Province population, number of employees and number of establishments are taken as inputs and number of insured and retired are taken as outputs. The data belong to 2012 and is taken from the website of SII.

The first level efficient DMU's (the ones that met $\theta_k^1 = 1$) are obtained by solving model (1) for $l = 1$ firstly since model (1) provides stratification of DEA. If looked more carefully, model (1) becomes input oriented CCR model when $l = 1$. Since we used input oriented model, DMU's with the objective function value of 1 are efficient. The first level efficiency limit (E^1) is also the original DEA limit and includes radial efficient DMU's.

$$E^1 = \{34,41,59,67,72,73,74\}$$

Then the DMU's in E^1 are removed from J^1 and

$$J^2 = \left\{ \begin{array}{l} 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25 \\ 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50 \\ 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 75 \\ 76, 77, 78, 79, 80, 81 \end{array} \right\}$$

series is obtained. If model (1) is solved for $l = 2$ for DMU's in J^2 , the second level efficient DMU's (the ones that met $\theta_k^2 = 1$) are obtained.

$$E^2 = \{06, 11, 16, 23, 43, 47, 63, 78, 81\}$$

Therefore we find the efficiency limit of J^1 as E^1 and the efficiency of J^2 as E^2 .

By repeating this process, we obtain the DMU subseries and their efficiency limits. This algorithm is continued until there are no more inefficient DMU's in the obtained sub series.

Finally, we obtain DMU's, $E^9 = \{13, 62, 75\}$, in the 9th level efficiency level. Therefore, DMU's are divided by efficiency limit in 9 level ($L = 9$). Efficiencies of DMU's based on radial measurement and their levels are as below:

Table 4.2. Efficiency scores and their levels

PROVINCES	EFFICIENCY SCORES	LEVEL	PROVINCES	EFFICIENCY SCORES	LEVEL
Adana(01)	0.7136	4	Konya(42)	0.6150	6
Adiyaman(02)	0.6981	7	Kütahya(43)	0.9638	2
Afyonkarahisar(03)	0.6716	6	Malatya(44)	0.8083	4
Ağrı(04)	0.7624	5	Manisa(45)	0.7544	4
Amasya(05)	0.6947	6	Kahramanmaraş(46)	0.8694	3
Ankara(06)	0.8647	2	Mardin(47)	0.9197	2
Antalya(07)	0.7393	3	Muğla(48)	0.6887	4
Artvin(08)	0.7450	4	Muş(49)	0.7964	4
Aydın(09)	0.7505	4	Nevşehir(50)	0.6578	6
Balıkesir(10)	0.8082	3	Niğde(51)	0.5962	8
Bilecik(11)	0.9196	2	Ordu(52)	0.7073	4
Bingöl(12)	0.8823	3	Rize(53)	0.7768	3
Bitlis(13)	0.5530	9	Sakarya(54)	0.8739	3
Bolu(14)	0.7713	4	Samsun(55)	0.6588	6
Burdur(15)	0.5931	7	Siirt(56)	0.8125	4
Bursa(16)	0.9467	2	Sinop(57)	0.6979	4
Çanakkale(17)	0.6477	5	Sivas(58)	0.7650	4
Çankırı(18)	0.7739	5	Tekirdağ(59)	1	1
Çorum(19)	0.7105	5	Tokat(60)	0.67	6
Denizli(20)	0.7790	3	Trabzon(61)	0.6989	5
Diyarbakır(21)	0.8059	4	Tunceli(62)	0.6045	9
Edirne(22)	0.6245	6	Şanlıurfa(63)	0.9522	2
Elazığ(23)	0.8667	2	Uşak(64)	0.7114	5
Erzincan(24)	0.7762	5	Van(65)	0.9081	3
Erzurum(25)	0.7280	6	Yozgat(66)	0.6298	8
Eskişehir(26)	0.8831	3	Zonguldak(67)	1	1
Gaziantep(27)	0.7536	3	Aksaray(68)	0.6560	7
Giresun(28)	0.6576	4	Bayburt(69)	0.6789	8
Gümüşhane(29)	0.6726	8	Karaman(70)	0.9316	3
Hakkari(30)	0.7302	5	Kırıkkale(71)	0.7084	4
Hatay(31)	0.6652	6	Batman(72)	1	1
Isparta(32)	0.61	7	Şırnak(73)	1	1
Mersin(33)	0.6562	5	Bartın(74)	1	1
İstanbul(34)	1	1	Ardahan(75)	0.6352	9
İzmir(35)	0.8678	3	İğdır(76)	0.7282	6
Kars(36)	0.6756	7	Yalova(77)	0.8029	4
Kastamonu(37)	0.6513	5	Karabük(78)	0.8855	2
Kayseri(38)	0.7219	5	Kilis(79)	0.6746	8
Kırklareli(39)	0.7378	4	Osmaniye(80)	0.7925	4
Kırşehir(40)	0.6820	7	Düzce(81)	0.9758	2
Kocaeli(41)	1	1			

The provincial directorates with the lowest efficiency, Bitlis (13), Tunceli (62), Ardahan (75), are placed in the last level of efficiency limit (9th level). If looked carefully, efficiency level do not follow the ranks of the efficiency scores. For example, Yozgat (66) provincial directorate in the level 8 has more efficiency score than Konya (42) provincial directorate in the level 6.

The following table shows the DMU's and radial measurement intervals in different efficiency levels.

Table 4.3. Radyal ölçüme göre seviyeler

Levels	Provincial Directorate	Efficiency Interval
Level 1	34, 41, 59, 67, 72, 73, 74	1
Level 2	6, 11, 16, 23, 43, 47, 63, 78, 81	0,864 - 0,975
Level 3	7, 10, 12, 20, 26, 27, 35, 46, 53, 54, 65, 70	0,739 - 0,931
Level 4	1, 8, 9, 14, 21, 28, 39, 44, 45, 48, 49, 52, 56, 57, 58, 71, 77, 80	0,657 - 0,812
Level 5	4, 17, 18, 19, 24, 30, 33, 37, 38, 61, 64	0,647 - 0,776
Level 6	3, 5, 22, 25, 31, 42, 50, 55, 60, 76	0,615 - 0,7282
Level 7	2, 15, 32, 36, 40, 68	0,593 - 0,698
Level 8	29, 51, 66, 69, 79	0,596 - 0,678
Level 9	13, 62, 75	0,553 - 0,635

Let us examine the attractiveness scores of DMU's in the first two levels (E^1 and E^2). In order to do this, we need to take DMU's, which perform worse than the level of DMU's in interest, as evaluation context. After taking these DMU's, the results obtained from the solution of model (2) is as following:

Table 4.4. Attractiveness scores for the province directorates in the first and second level

Level	Province Directorate	Evaluation Context							
		E^2	E^3	E^4	E^5	E^6	E^7	E^8	E^9
L E V E L 1	34	1,2219	1,4014	1,6016	1,7441	2,0157	1,2975	3,0529	3,7393
	41	1,1661	1,3822	1,5413	1,6703	1,8913	2,1907	2,9144	3,7339
	59	1,3726	1,5261	1,7208	2,0978	2,2379	2,696	3,6421	4,3896
	67	1,4781	1,5283	1,7045	2,1205	2,1506	2,6118	3,8956	6,5353
	72	1,1385	1,2038	1,2712	1,3669	1,5223	1,5675	1,746	1,8713
	73	1,2495	1,3335	1,3688	1,4284	1,678	1,818	1,955	2,054
	74	1,2965	1,5081	1,6907	1,9216	1,9894	2,3714	3,734	6,2774
L E V E L 2	6	1	1,1227	1,2181	1,4136	1,6734	1,7091	2,1879	2,8388
	11	1	1,1028	1,2289	1,3036	1,5304	1,5921	2,0891	3,4848
	16	1	1,2142	1,3801	1,5377	1,6757	2,0247	2,6596	4,2833
	23	1	1,0817	1,1537	1,2388	1,288	1,4482	1,8009	2,7079
	43	1	1,3104	1,4745	1,6357	1,7731	2,062	3,1839	5,302
	47	1	1,0697	1,2094	1,3135	1,4488	1,4984	1,7065	1,8647
	63	1	1,117	1,2855	1,4524	1,5755	1,6075	2,1326	2,4971
	78	1	1,2156	1,3525	1,5973	1,7675	2,0458	3,0968	5,1857
	81	1	1,2612	1,3152	1,5254	1,6225	1,9533	2,6357	3,3997

When evaluation contexts are selected as E^2 and E^3 , Zonguldak (67) provincial directorate in the E^1 level has the highest attractiveness scores. Greater the attractiveness score ($\theta_q^d > 1$), the more attractive the DMU in interest is.

If looked carefully, the attractiveness scores, therefore performances, are changes when evaluation contexts are changed. For example, the most attractive provincial directorate is Tekirdağ (59) with the evaluation context, E^4 , and Zonguldak (67) with E^8 for DMU's in the first level.

When the results for DMU's in E^2 level are examined, only one provincial directorate has the highest attractiveness when different evaluation contexts (E^3, E^4, \dots, E^9) are considered and that province is Kütahya

(43). This means that Kütahya (43) provincial directorate proves itself to be more unique in the evaluation contexts.

When E^7 is considered as the evaluation context, if we sort the attractiveness scores of DMU's in the E^2 we obtain the following:

Kütahya > Karabük > Bursa > Düzce > Ankara > Şanlıurfa > Bilecik > Mardin > Elazığ

And now, let us look at the progress scores of DMU's in the second level. In order to do this, DMU's in the levels, which perform better than DMU's in E^2 , are taken as evaluation context. After doing this, solving model (3) gives the following results:

Table 4.5. Progress scores for province directorates in the second level

Province Directorates	Evaluation Context
	E¹
06	0,8647
11	0,9196
16	0,9467
23	0,8667
43	0,9638
47	0,9197
63	0,9522
78	0,8855
81	0,9758

When the results are examined, the province directorate with the highest progression in E^2 level is Ankara (06). The smaller progression score, the more progression ($\varphi_q^g < 1$) will have in DMU in interest. When we sort province directorates based on their progressions with regard to E^1 evaluation context, which is the only level with better performance than itself for E^2 level, we obtain the following:

Ankara > Elazığ > Karabük > Bilecik > Mardin > Bursa > Şanlıurfa > Kütahya > Düzce.

Finally, let us examine the reference set of DMU's in a specific level.

From the definition of reference set, $R_k^{CCR}(l) = \{j \in J^l | \lambda_j > 0\}$, DMU's with $\lambda_j > 0$ density vector that is calculated from model (2) are in the reference set of DMU in interest. For example, reference sets for DMU's in the 9th efficiency limit (E^9) are as below:

Table 4.6. Reference sets as comparison targets for province directorates in the 9th level

Province Directorates	$R^{CCR}(1)$	$R^{CCR}(2)$	$R^{CCR}(3)$	$R^{CCR}(4)$	$R^{CCR}(5)$	$R^{CCR}(6)$	$R^{CCR}(7)$	$R^{CCR}(8)$
13	59, 72, 73	47, 81	12, 65, 70	21, 56	4, 18, 38	3, 25, 76	2, 68	66, 79
62	59, 67, 72	23, 47, 81	12, 53, 70	44, 56, 58	4, 18, 24	5, 25, 50	2, 40, 68	69, 79
75	59, 67, 72	23, 47, 81	12, 53, 65, 70	21, 44	4, 18	25, 76	2, 40, 68	29, 66, 79

As we can see from Table 4.6, while suitable comparison targets in the 1st level for Bitlis (13) province directorate are Tekirdağ (59), Batman (72) and Şırnak (73), in the 4th level, they are Diyarbakır (21) and Siirt (56), and in the 8th level, Yozgat (66) and Kilis (79).

Therefore, for the Bitlis (13) province directorate, which is in the last level, comparison targets are provided starting from the levels with the closest performances ($E^8, E^7, E^6, \dots, E^1$) to itself step by step and reference sets are formed from these efficient province directorates. This way, Bitlis (13) province directorate had the opportunity to progress step by step to improve their performance.

4. Results

After the efficiency limit is defined in the original DEA method, addition or elimination of inefficient DMU's does not change the efficiency limit or efficiencies of the current DMU's. However, in context-dependent DEA, this process changes the performances of both efficient and inefficient DMU's (Seiford and Zhu, 2003). This change makes context-dependent DEA versatile and enables to determine better alternatives locally and globally for DMU's. It can detect the prominent performances using attractiveness and progress measures and it is possible to see the difference between performances of efficient DMU's more easily compared to the original DEA.

As a result, the context-dependent DEA approach, which is an extension of DEA, can obtain more positive approaches that will guide the researchers about the performances of DMU's, make comparisons in detail and create forward looking strategic targets in a more efficient way.

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