*International*
*Journal*
*Of Advanced Research in Engineering& Management (IJAREM)*
*ISSN: 2456-2033 || PP. 26-31*

# Research of the Influence of Design Features of the Toothed Operating tool on the Tillage Efficiency

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**Abstract:** There has been proposed a mathematical model which, under certain conditions, will allow the development of a toothed tool with such a shape of the working surface that will satisfy the agro technical, technological and economic parameters during soil cultivation.

The operating tool is made in the form of a block of teeth. The valley and the protrusion of the tooth in the horizontal plane are made along a logarithmic spiral, the protrusions are made in the form of the fourth degree parabola.

The toothed tool equation is presented in the form of a combination of rotation, displacement and compression matrices.

The working surface is given kinematically as the trajectory of motion of the points of the generating logarithmic spiral.

Equations obtained describe the surface of the operating tool in the areas of valleys and protrusions.

**Keywords:** toothed operating tool, soil, tooth protrusions and valleys, logarithmic spiral, fourth order parabola, mathematical model, matrix-vector solution.

The task of designing a toothed operating tool for tillage machines is reduced to the choice of a mathematical model that would meet all the demands made to the operating tool in the technological, economic terms, considering the physical and mechanical properties of the soil.

The operating tool is made in the form of a block of teeth (Fig. 1) constituting valleys in the horizontal plane made along a logarithmic spiral, and protrusions profiled along the fourth degree parabola.

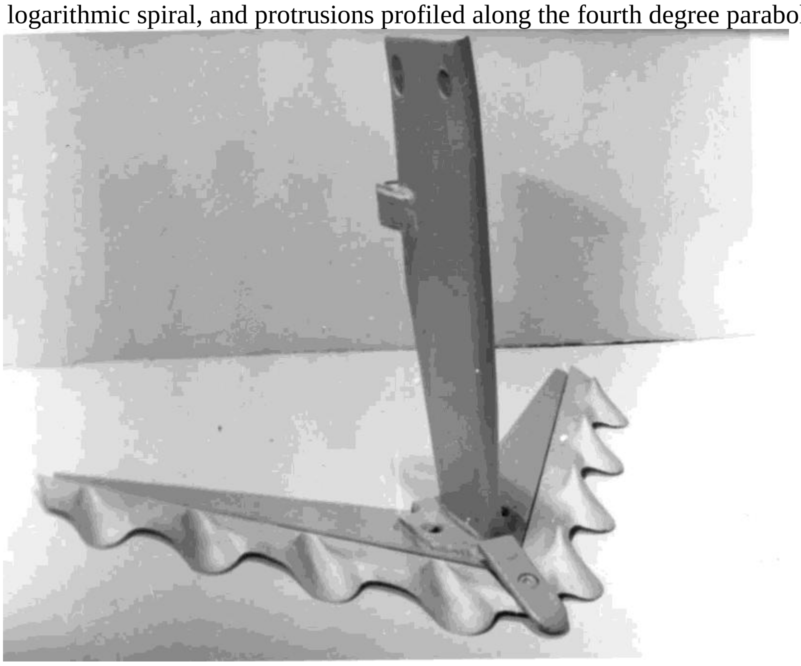

Figure 1 – General view of the toothed operating tool for tillage.

In horizontally projecting planes, the section of the tooth is a family of logarithmic spirals

$$r_{\psi_i} = r_0 e^{\psi tg\phi}, \qquad (1)$$

where $r_{\psi_i}$ is current radius vector;

$r_0$ is initial radius vector;

$tg\phi$ is internal friction coefficient;

$\psi$ is the current angle of the radius vector of the spiral.

The equation of the surface of the toothed operating tool for tillage can generally be represented using the matrix

$$A_{ij} = \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ \hline 0 & 0 & 0 & 1 \end{array}\right), \qquad (2)$$

where $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is third-order matrix describing rotation in three-dimensional space;

$(a_{14}, \rightleftarrows a_{24}, a_{34})^T$ are translational components.

The working surface will be set kinematically as trajectories of points of the generating logarithmic spiral (Fig. 2) located in the plane $OX_2X_3$ and performing rotational and translational movements.

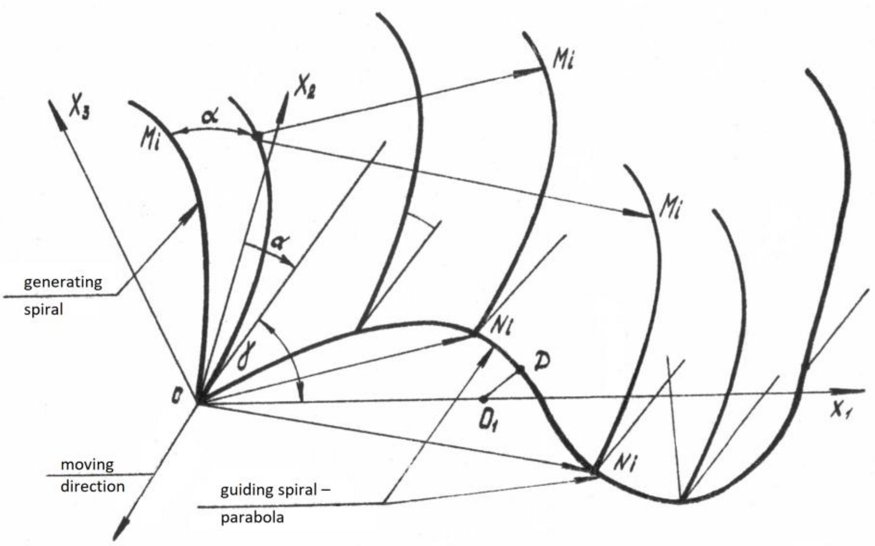


Figure 2 – Kinematic model of the surface of the operating tool.

The rotation of the points $M_i$ of the spiral around the axis $OX_3$ at the angle α = 90-γ (where γ is the opening angle of the paw of the operating tool) is described by the rotation matrix

$$A_r = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (3)$$

The coordinates of the current points $M_i$ of the generating logarithmic spiral, determined by equation (1), where $\varphi = 45^0$ and $\psi = -45^0 ... 50^0$ will have the following values (Fig. 3)

$$x_1^G = 0, \; x_2^G = r_{\psi_i} \sin \psi_i \;\; ; \quad x_3^G = r_{\psi_i} \cos \psi_i .$$

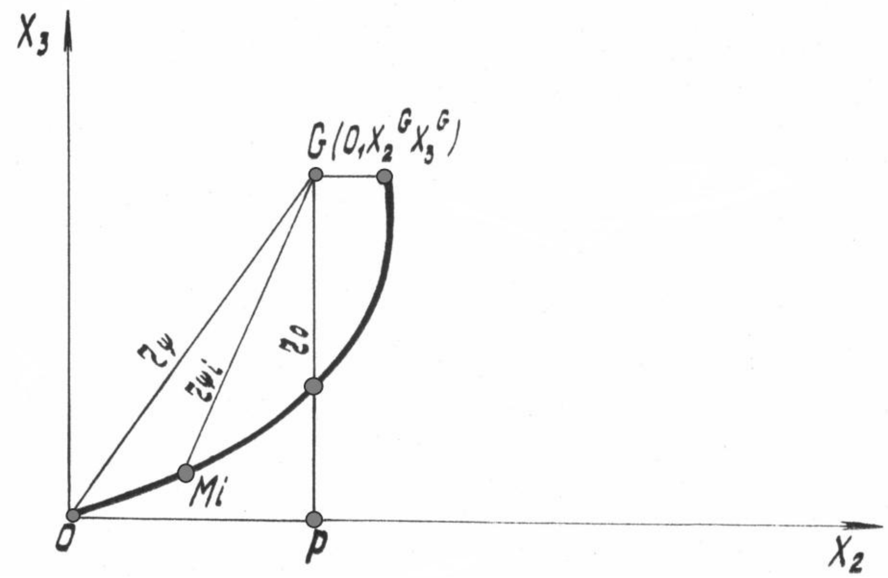

Figure 3 – Type of the generating spiral.

The coordinates of the current points $M_i$ of the generating spiral in Cartesian coordinates, depending on $\psi$ will be written as follows

$$x_1^{r\psi_i} = 0;$$
$$x_2^{r\psi_i} = OP - r_{\psi_i} \sin \psi_i \, ; \qquad (4)$$
$$x_3^{r\psi_i} = GP - r_{\psi_i} \cos \psi_i .$$

The translational movement of points $M_i$ in area of valley or protrusion is described by the displacement matrix.

$$A_n = \begin{pmatrix} 1 & 0 & 0 & \Delta x_1^{N_i} \\ 0 & 1 & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (5)$$

where $\Delta x_1^{N_i}$, $\Delta x_2^{N_i}$, $\Delta x_3^{N_i}$ are vector components.

Thus, the total displacement of the points of the generating logarithmic spiral is determined by scalar matrix multiplication

$$A = A_n \cdot A_r = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & \Delta x_1^{N_i} \\ \sin\alpha & \cos\alpha & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (6)$$

Then the equation of the surface of the toothed operating tool can be represented as

$$Y_i = A \cdot X_i, \qquad (7)$$

where $X_i$ are coordinates of the current point $M_i$ on the generating spiral, determined by the equations(4); $Y_i$ are coordinates of points on the surface of the toothed operating tool depending on the parameter $\psi_i$, which is the generating logarithmic spiral, and the magnitude of the vector $ON_i$ determined by the position of the current point $N_i$ on the valley or protrusion of the cutting blade edge.

For the valley area the coordinates of the current point $N_i$ of the vector of displacement along a logarithmic spiral (Fig. 4) are determined by the following equations

$$x_1^{N_i} = x_1^{O_1} + r_{\theta_i} e^{\theta_i tg\phi} \cos \bar{\theta_i};$$
$$x_2^{N_i} = r_{\theta_i} e^{\theta_i tg\phi} \sin \bar{\theta_i}; \qquad (8)$$
$$x_3^{N_i} = 0,$$

where $\theta_i$=180° are initial angles;

$\bar{\theta_i} = 20^o + \theta_i$ are coordinates of point $O_1(^S/_2, 0{,}0)$;

$S$ is tooth pitch.

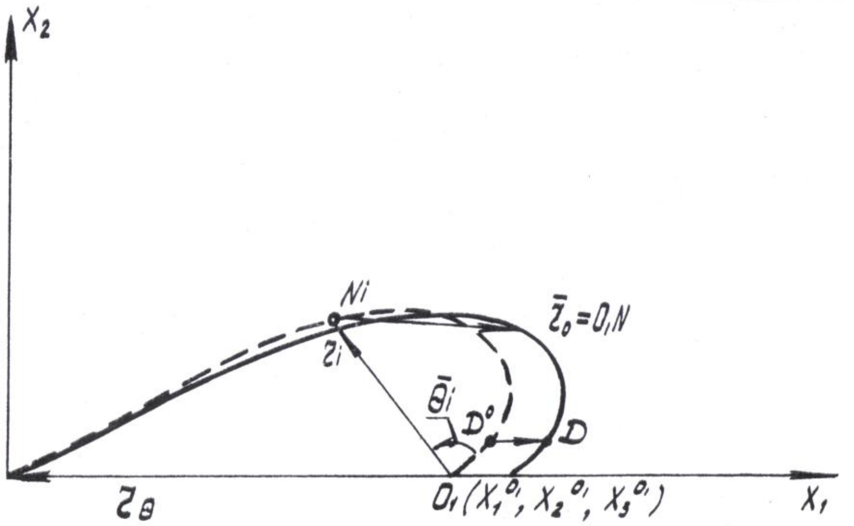


Figure 4 – Valley spiral transformations.

In connection with the lengthening of the logarithmic spiral of the valley profile along the axis $OX_1$, the coefficient is introduced

$$\kappa = \frac{1}{\cos\alpha},$$

while the transformation of the elongation is written by the matrix

$$A_k = \begin{pmatrix} \kappa & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (9)$$

Considering equation (9), the coordinates of the points of the translational displacement vector can be determined as

$$\Delta x_i = A_k \cdot x_i;$$

$$\Delta x_1^{N_i} = \kappa x_1^{N_i}; \qquad (10)$$

$$\Delta x_2^{N_i} = x_2^{N_i};$$
$$\Delta x_3^{N_i} = 0.$$

For the protrusion area the coordinates of the current point $N_i$(Fig. 5) of the vector of translational displacement along the parabola $x_2 = x_1^4$ are determined as the product of the following transformations:

a) compression along the axis $OX_2$ with coefficient $\mu$, described by the matrix

$$A_p = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \qquad (11)$$

b) elongation along the axis $OX_1$ with coefficient $k$,described by the matrix (9);

c) translational displacement along the vector $CD$,described by the matrix (5).

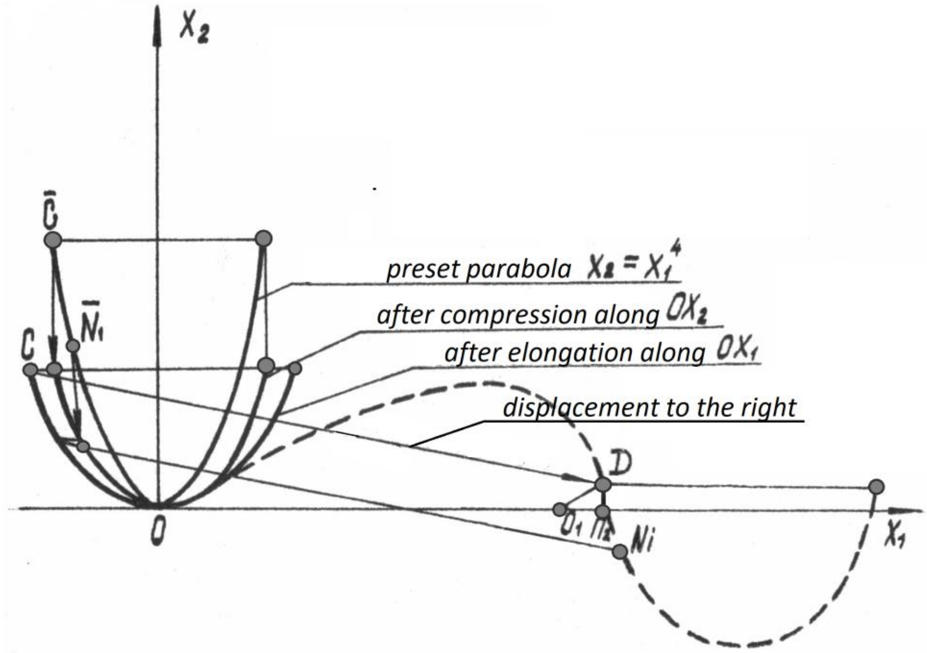


Figure 5 – Coordinates of the current point $N_i$ of the parabola in the place of protrusions.

The general transformation of the parabola is then described by the matrix

$$A_z = A_n \cdot A_k \cdot A_p;$$

$$A_z = \begin{pmatrix} k & 0 & 0 & \Delta x_1^{CD} \\ 0 & \mu & 0 & \Delta x_2^{CD} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (12)$$

The coordinates of the current point $N_i$ on the parabola will look like this in matrix form

$$\Delta x_i = A_z \cdot x_i, \qquad (13)$$

Where $X_i$ are coordinates of points of the original parabola $x_2 = x_1^4$;

$\Delta x_i$ are coordinates of the points of the transformed parabola which are defined as follows:

$$\Delta x_1 = kx_1 + \Delta x_1^{CD};$$
$$\Delta x_2 = \mu x_2 + \Delta x_2^{CD}; \qquad (14)$$
$$\Delta x_3 = 0.$$

The equation of the surface of the toothed operating body (7) in matrix form can be represented as

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & \Delta x_1^{N_i} \\ \sin\alpha & \cos\alpha & 0 & \Delta x_2^{N_i} \\ 0 & 0 & 1 & \Delta x_3^{N_i} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1^{M_i} \\ x_2^{M_i} \\ x_3^{M_i} \\ 1 \end{pmatrix}, \qquad (15)$$

Where $x_1^{M_i}, x_2^{M_i}, x_3^{M_i}$ are coordinates of the points of the generating spiral; $\Delta x_1^{N_i}, \Delta x_2^{N_i}, \Delta x_3^{N_i}$ are coordinates of the points of the cutting blade.

This equation describes the surface of the tooth in the places of valleys and protrusions.

Finite formula for the coordinates of the tooth surface points for the valley area can be put down as follows

$$\begin{aligned} & y_1 = \sin\alpha\left(OP - r_{\psi_i}\sin\psi_i\right) + k\left({}^{s}/_{2} + r_{\theta_i}\cos\bar{\theta}_i\right); \\ & y_2 = \cos\alpha\left(GP - r_{\psi_i}\sin\psi_i\right) + r_{\theta_i}\sin\bar{\theta}_i; \qquad (16) \\ & y_3 = GP - r_{\psi_i}\cos\psi_i, \end{aligned}$$

and for the protrusion area in the following way:

$$\begin{aligned} & y_1 = \sin\alpha\left(OP - r_{\psi_i}\sin\psi_i\right) + kx_i + \Delta x_1^{CD}; \\ & y_2 = \cos\alpha\left(GP - r_{\psi_i}\sin\psi_i\right) + \mu x_2 + \Delta x_2^{CD}; \qquad (17) \\ & y_3 = GP - r_{\psi_i}\cos\psi_i. \end{aligned}$$

## Conclusion

The obtained mathematical model (16), (17) for description of a toothed operating tool surface makes it possible, under various conditions, to obtain a family of toothed tillage tools, which will be workable in specific conditions, while taking into account the depth of tillage, agro technical requirements for the quality of crumbling soil, physical and mechanical properties of the soil, geometry of the cutting edge and the number of teeth. The shapes of the surfaces of the operating tools depend on the values of functional parameters $k, \mu, \theta, \psi$.