

## **Mathematical Modeling, Design and Control of Hexacopter Maneuvers using Matlab Simulink Platform**

<sup>1</sup>Kirandeep Kaur, <sup>2</sup>Er. Gurjit Singh

<sup>1</sup>Research Scholar, <sup>2</sup>Assistant Professor,

<sup>1</sup>Amritsar Group of Colleges, Amritsar, Punjab

<sup>2</sup>Amritsar Group of Colleges, Amritsar, Punjab

---

**Abstract:** This work establishes the kinematics model of hexarotor helicopter by Newton Euler method, and obtains rotor lift model through experiment. For attitude and displacement control (roll, pitch and yaw maneuver's) of hexarotor helicopter is achieved by adaptive control-optimization method in practical systems. Simulink models are generated for the control purpose. These are either models describing the physical system components for Simscape simulations or mathematical models of hexacopters for model validation or model based control design purposes. Other files involve environmental and gravitational effects in the simulation environment, system parameters and plotting functions. Simulink is an extension of Matlab that allows for rapid and accurate building of computer models of dynamical systems using diagram notation. Using Simulink, new ideas are often easily integrated and tested immediately. Results of simulation and experiment show that proposed method can meet the stability and rapidity requirement of hexarotor helicopter control and has better robustness and real-time performance. Altitude, roll, pitch and yaw trajectories of the proposed model has been compared with the real time trajectories in which one has to maneuver in order to provide translational and altitude movements to the copter. It has been found that the proposed set up follows the desired trajectory with minor harmonic fluctuations when there is a shift from one maneuver to another.

**Keywords:** Roll-pitch-yaw control, hexacopter, matlab, solidworks, Trajectory control etc.

---

### **I. Introduction**

An Unmanned Aerial Vehicle (UAV), commonly known as drone, is an aircraft characterized by the absence of the human pilot on board. Its flight is controlled by the on board system, under the remote control of a navigator or pilot on the ground. Anyhow, the flight operations of an UAV must comply with the same rules and procedures of the aircraft with on board pilot and flight crew. The use of UAVs is well established for military purposes but also for civil applications, such as in fire prevention and emergency response operations, for nonmilitary security, in pipelines surveillance, for remote sensing and research aims, and, more in generally, in all cases in which such systems may allow the execution of "dull, dirty and dangerous" missions, often at a lower cost than conventional aircraft. These research fields are multidisciplinary and involves areas like aeronautics, computer science, mathematics, electronics, mechanics, automatic control, signal processing, etc. An UAV is also able to travel places where pilots cannot go or where there is high risk involved, for example in disaster areas after nuclear blasts, areas with unstable weather, volcanoes, etc..Also tedious aerial surveillance can be made fully automatic so that fewer operators are needed. In any case, it is necessary to note that an UAV with a payload can be an expensive piece of equipment and therefore high demands are set on the automatic flight control system.

Moreover, in harsh environments failure situations can arise and in these cases the control system should be able to detect and counteract these failures preventing the aircraft from crashing. It is clear that the development of a self-learning and self-adaptive flight controller system is a primary step in the Unmanned Aerial Vehicles, in order to manage the aircraft when flight conditions change, while, on the contrary, traditional flight control systems remain unchanged after design. A. Flight Control Systems of UAVs A key aspect of the UAVs is related to their control systems. In recent years, several nonlinear control methods, such as the Lyapunov function [1], back-stepping [2] and nonlinear dynamic inversion [3], have been applied in small UAV flight control systems. Back-stepping techniques can derive air speed and roll control commands from known heading and air speed control laws which explicitly account for the heading rate and air speed constraints of the UAV.

The nonlinear dynamic inversion uses a system model in order to control it and to eliminate the need for gain scheduling improving, at the same time, the performance. Several techniques for nonlinear dynamic

inversion flight control laws are based on linear methods. Specifically, these techniques are applied to linear systems which are obtained by linearizing the nonlinear systems. However, the main disadvantage of nonlinear dynamic inversion is that it strongly depends on precise nonlinear models, since the innate character of nonlinear dynamic inversion remove the nonlinearities. Whereby this limit the performance when applied to systems which only have approximate nonlinear models as UAVS. The above methods improve the attitude and trajectory control performance of UAVs, however, the presented control methods are too complex to design. Other new approaches are based on soft computing techniques whose objective is to evaluate, decide, monitor and measure in a unclear and vague field emulating and using humans ability to perform the above activities on the basis of experiences. In fact, in recent years, soft computing techniques are applied in various research fields such as road monitoring [4], home automation [5], smart localization [6], energy savings [7] and industrial networks [8]; moreover, they found also applications in aerospace [9].

Hexacopter has six-rotors located on the vertices of a regular hexagon and the propulsion system consists of three pairs of counter-rotating fixed-pitch propellers. In order to build an appropriate mathematical model the hexacopter is assumed as a rigid body, thus its dynamics can be studied by means of Newton-Euler differential equations, which consider both internal and external influences that affect the multirotor. A robust mathematical modeling together with efficient control system guarantees an easy flight management of the multicopters. Therefore main focus in this work is to implement the control system of a hexacopter in terms of output speed needed to flight and turn the hexacopter into a particular desired direction. The Proportional Integrative Derivative (PID) Controller, PD controller and PI controllers can be considered in this work so that pre-defined altitude, yaw, pitch and roll inputs can be followed and by this effectiveness of actual output can be analyzed for individual roll, yaw, pitch and altitude variations in order to get the desired performance to fly the multicopter on a particular trajectory.

## II. Literature Review

**Alaimo, A. et al. (2013) [10]** presented a mathematical model of a mini rotorcraft with six rotors and its control. The equations of motion has been defined by means of quaternions since, unlike Euler angles, they do not suffer from the gimbal lock; they are also more efficient in terms of numerical computation and, in addition, any operations involving them is trivial. It has been also supposed that in real applications the drone will only assume configurations far from the gimbal lock. However, quaternion parametrization is taken into account because of its simplicity for computation and its numerical stability, that allows more efficient and fast algorithm implementation with higher control system. A PD controller based on quaternions has been then implemented by an innovative quaternion error definition. Numerical results on trajectory control has been performed and discussed. Future works will extend in several directions. The model presented in this paper is a simplification of more complex dynamics, indeed aerodynamical effects have been neglected. Thus, next step will involve the improving of the hexacopter model with more realistic features and the application of more accurate control laws. Moreover, an integrated term in PD controller will be introduced achieving a different PID controller, in order to be applied in real flights.

**Collotta, M. et al. (2014) [11]** presented a real-time system, based on a neural network model, in order to develop proper methods for stabilization and trajectory control of a hexacopter. The rapid response of this technique and the high quality of data approximation have been shown. Experimental results are very promising in terms of error measures and obtained coordinates of the hexacopter. The next step consists in the integration of the whole dynamical system, considering the forces and the moments acting on the UAV, in order to obtain the position and the orientation of the UAV together. Moreover, another objective will be the neural network optimization, for example through the analytical calculation of the embedding parameters.

**Gupta, A. K. et al. (2014) [12]** presented the design and fabrication of a hexa-copter along with its static and dynamic analysis. The ASHA robot is capable of carrying a payload of up to 3.4kg, a total flight time of up to 25 minutes through experiments, which is in line with the theoretical flight time of about 30 minutes. The direction and altitude are easily controllable using a Remote controller due to aid of onboard flight controller and feedback system. The sturdy design of robot enlarges the scope of future adaptation and development in regards with the application in various fields such as defence, surveillance, and rescue operations.

**Alaimo, A. et al. (2014) [13]** presented an efficient and reliable PID technique based on quaternion for a hexacopter control and stabilization and discussed by means of a wide experimentation. The PID controlled has been applied to Newton-Euler equations, describing the dynamical behaviour of the hexacopter. Numerical results emphasized that actual position gained by the numerical integration of the dynamical system, overlaps the checkpoints fixed along the path of the desired position. Moreover, the required attitude is reached in few seconds and the hovering state is preserved during the entire flight. The PID controller parameters do not vary with the desired position. These promising results induce to an implementation of the presented technique in real flights, where rapid and reliable responses of the controller are necessary to manoeuvre and manage the UAVs.

**Criado et al. (2015) [14]** described the design of methodology for functional autopilot for UAV or quadrotor AR. Drone. This is a self-stabilized platform and is developed by Parrot. The main goal is to design a control strategy for autonomous path tracking in the XY plane, comparing two different control techniques. Three phases are carried out to achieve this objective: model identification, control design and evaluation, and implementation. The two different techniques to reduce path tracking error are compared. Both techniques, the speed limiter strategy and point generator does not correct the path, but provides better time performance quad copter. This is resulting that Computer vision based strategies using the frontal camera could make it possible to add more capabilities to the auto pilot.

**Martinez et al. (2015) [15]** presented a hierarchical two-level control strategy for the trajectory tracking of the quadcopter. They design external loop using variable substitution to move the translational dynamics of the robot to follow smooth parametric functions. The outer loop generates the desired trajectories of the orientation angles, which are controlled in the inner loop. They calculate parameter gains via linear optimal control in order to minimize the energy consumption of the rotors. Some numerical simulations show the performance of the control approach.

**Panizza et al. (2015) [16]** consider various problems of characterizing the attitude dynamics of variable pitch quadrotor and they also identify various approaches to collect the data of real quadrotor. They observe the performance and collect data even on-line and off-line methods by comparing under black-box and grey-box models. This results the subspace approach to be good option to identification part of fast and high automated control design tool chain for quadrotor attitude.

In the research work, the mathematical model as well as the PID controller, as a generalization of the Proportional Derivative (PD) controller explained in [18], will be implemented and then simulated using Simulink MATLAB software. We decided to design the 3D model of hexacopter in CAD software (SOLIDWORKS). Then MATLAB Simulink will be used to simulate the behavior of this model from the control framework that will be developed in order to move this in a desired directory as SOLIDWORKS assembly models can be called in SIM-MECHANICS TOOLBOX of the SIMULINK MATLAB. It is easier to check the performance without need to make a practical prototype model. Or in other ways, if one wants to build it, it is better to simulate in this way so that same operations can be gained from the actual controller that can be used while building the actual model. So from above discussion some objectives have been formulated in order to measure the effectiveness of the proposed work.

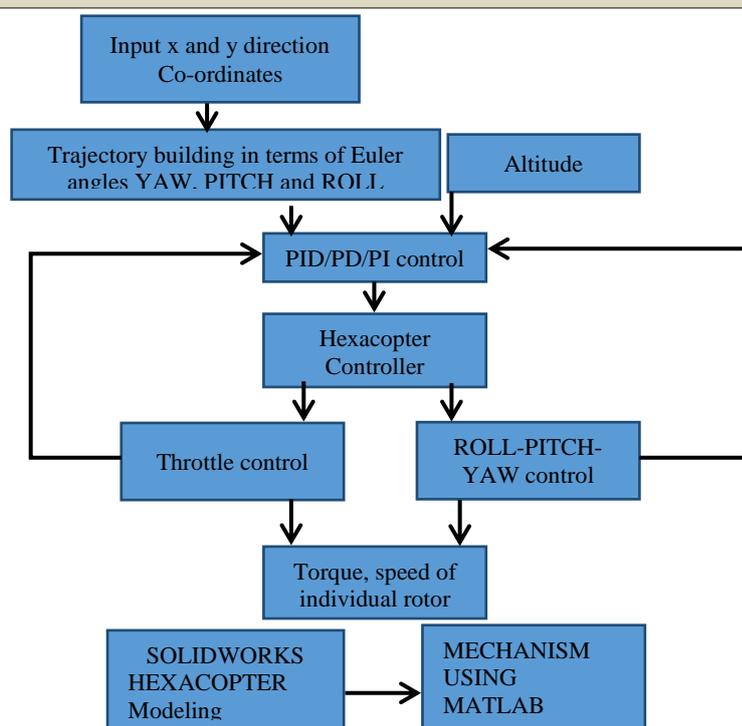


Figure 1: Methodology of proposed work

### III. Proposed work

The inertial frame is denoted as  $R_I$  and the Body fixed frame on the hexacopter body is denoted as  $R_B$ . The three Euler Angles, namely yaw angle  $\varphi$ , pitch angle  $\theta$ , roll angle  $\phi$  are used to describe the orientation of the hexacopter [19]. Together all three angles form the vector  $\eta = [A\phi \theta \varphi]^T$ . The range of angles  $\phi, \theta$  is  $(-\frac{\pi}{2}, \frac{\pi}{2})$  and for  $\varphi$  is  $(-\pi, \pi)$ . The hexacopter performs the change in three Euler angles assuming that the origin is the same before and after all the changes. The inertial frame is transformed to the body fixed frame by using rotation matrices. To generate the final inertial frame the order of rotation is implemented in sequence [19]. The sequence starts with rolling about X-axis to the new frame  $R_{B1}$ , by the roll angle  $\phi$ . Followed by pitching about Y-axis to the next new frame  $R_{B2}$ , by the pitch angle  $\theta$ . At last yawing about Z-axis to the frame coincided with inertial frame  $R_I$ , by the yaw angle  $\varphi$ .

The total rotational matrix is obtained by multiplying  $R(x, \phi)$ ,  $R(y, \theta)$  and  $R(z, \varphi)$  which is denoted by  $R_L$ .

$$\begin{bmatrix} x_B \\ y_B \\ z_B \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \varphi & \cos \theta \sin \varphi & -\sin \theta \\ \sin \phi \sin \theta \cos \varphi - \cos \phi \sin \varphi & \sin \phi \sin \theta \sin \varphi + \cos \phi \cos \varphi & \sin \phi \cos \theta \\ \cos \phi \sin \theta \cos \varphi + \sin \phi \sin \varphi & \cos \phi \sin \theta \sin \varphi - \sin \phi \cos \varphi & \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix} \quad \dots 4$$

The forces acting on the hexacopter are gravity, torque and air friction aerodynamics forces. The rotation of the propellers are the source of torque and thrust. The total thrust produced by the propellers is used against gravity and drive its motion in lateral plan.

The force of gravity is acting along Z-axis in the inertial frame. The contribution of gravitational force  $F_G^B$  in the body fixed frame is expressed using rotational matrix  $R_L$ ,

$$F_G^B = R_L \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = mg \begin{bmatrix} \sin \theta \\ \sin \phi \cos \theta \\ \cos \phi \cos \theta \end{bmatrix} \quad \dots 11$$

The sources of thrust are not at the center of gravity of hexacopter. The thrust generated by propellers will produce torques around axes of rotation. The geometry of the hexacopter is shown in Figure 2, in order to calculate the torque around X-axis and Y-axis for body fixed frame.

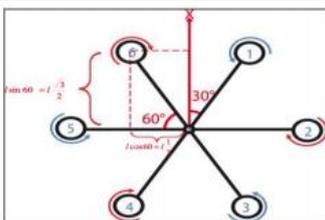


Figure 2: Hexacopter propellers distance from center of gravity and angles with X,Y-axes[17]

The numerical values of the parameters are from previous and similar projects [20].

Name	Description	Numerical value
m	Mass of platform	2.1 kg
l	Length of arm	0.32 m
$I_{xx}$	Moment of inertia around x-axis	$3.8 \times 10^{-3} \text{kgm}^2$
$I_{yy}$	Moment of inertia around y-axis	$3.8 \times 10^{-3} \text{kgm}^2$
$I_{zz}$	Moment of inertia around z-axis	$7.1 \times 10^{-3} \text{kgm}^2$
$C_T$	Propeller thrust constant	0.01458
$C_Q$	Propeller torque constant	$1.037 \times 10^{-3}$
R	Propeller radius	0.14m
$\rho$	Air density	$1.2 \text{kg/m}^3$
g	Acceleration of gravity	$9.81 \text{m/s}^2$

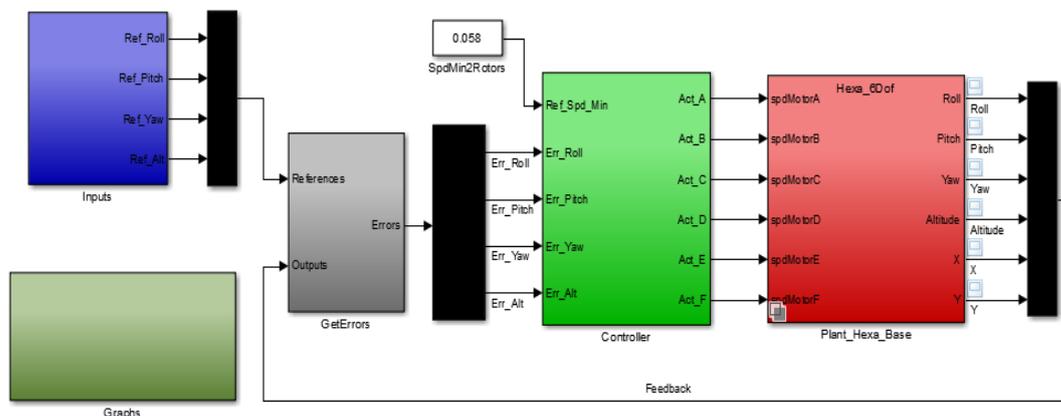


Figure 3: Proposed hexacopter system in MATLAB/SIMULINK

#### IV. Results and Discussions

Block diagrams of the developed hexacopter system model is shown in below figure. As it can be seen, it consists of the signal builder, error control, rotor speed control, feedback controllers, and part simulation.

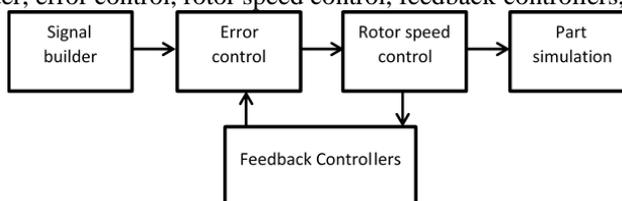


Figure 4: Proposed Hexacopter System

The Signal Builder block allows us to create interchangeable groups of piecewise linear signal sources and use them in a model. In this model, four waveforms are built each for altitude, roll, pitch and yaw testing. For checking the performance, all inputs are varied at different times to check the effect of each individual inputs. The outputs from this block are fed to error control block which compares the desired output needed generated by signal builder and the actual output of the rotor at that time and fed the error into controller block. The error block has been shown in figure 5 below.

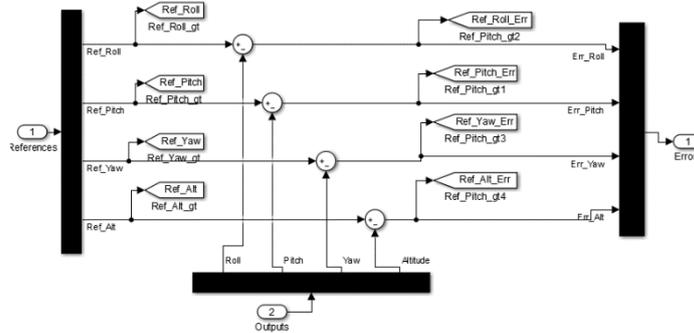


Figure 51: Error block

The minimum speed of the rotors is given to every rotor.

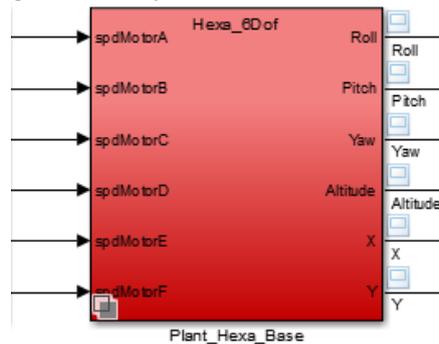


Figure 62: Hexa\_6Dof block

The inputs to this block is speeds calculated in the Controller block for each rotor. The output parameters given by this block is Roll, Pitch, Yaw and Altitude. The architecture of this block has two main blocks Base tests and Hexarotor.

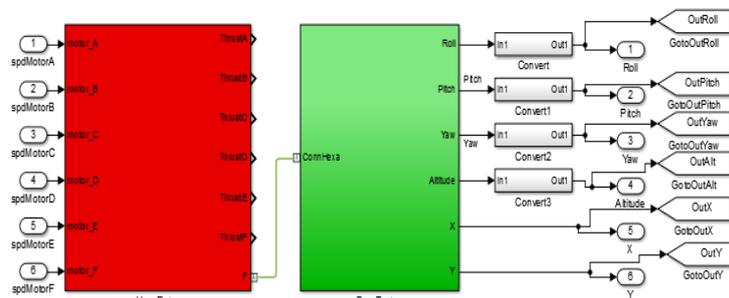


Figure 7: Architecture of Hexa\_6DOF

The input/ output plots for roll, pitch, yaw, altitude and motor speeds are shown below.

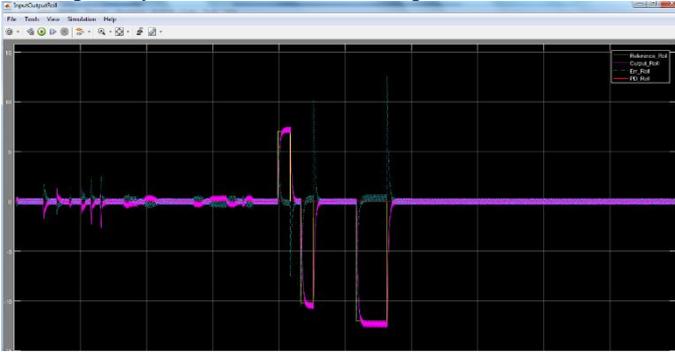


Figure 8: Input/ Output Roll



Figure 93: Input/ Output Yaw



Figure 104: Input/ Output Pitch

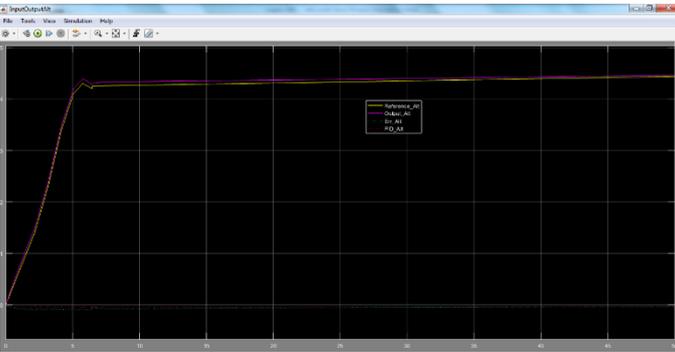


Figure 115: Input/ Output Altitude

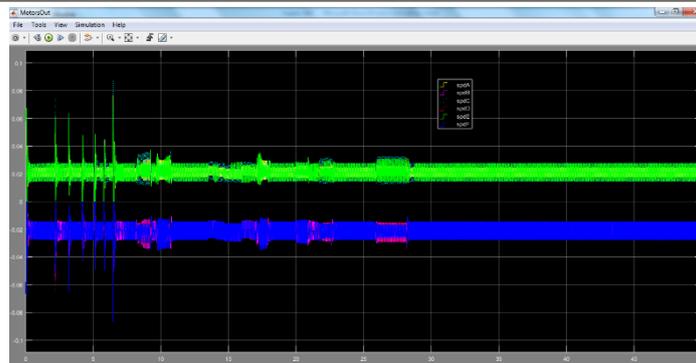


Figure 12: Motor speeds

## V. Conclusion

This work establishes the kinematics model of hexarotor helicopter by Newton Euler method, and obtains rotor lift model through experiment. For attitude and displacement control (roll, pitch and yaw maneuver's) of hexarotor helicopter is achieved by adaptive control-optimization method in practical systems. The 3D hexarotor parts has been designed in Solidworks software which are assembled as a steup using matlab's imscape library from Simulink. The generated matlab simulation files are combination of CAD models converted to a format compatible with Simscape simulation utility. Simulink models are generated for the control purpose. These are either models describing the physical system components for Sim scape simulations or mathematical models of hexacopters for model validation or model based control design purposes. Other files involve environmental and gravitational effects in the simulation environment, system parameters and plotting functions. Sim Simulink is an extension of Matlab that allows for rapid and accurate building of computer models of dynamical systems using block diagram notation. Using Simulink, new ideas can be easily integrated and tested immediately. Results of simulation and experiment show that proposed method can meet the stability and rapidity requirement of hexarotor helicopter control and has better robustness and real-time performance.

## References

- [1]. Hongda Chen, Kuochu Chang, C.S. Agate, UAV Path Planning with Tangent-plus-Lyapunov Vector Field Guidance and Obstacle Avoidance, IEEE Transactions on Aerospace and Electronic Systems, Vol. 49, Issue 2, pp. 840-856, 2013.
- [2]. R. Aruneshwaran, S. Suresh, J. Wang, T.K. Venugopalan, Neural adaptive flight controller for ducted-fan UAV performing nonlinear maneuver, IEEE Symposium on Computational Intelligence for Security and Defense Applications (CISDA), pp. 51-56, 2013.
- [3]. Zhufeng Xie, Yuanqing Xia, Mengyin Fu, Robust trajectory-tracking method for UAV using nonlinear dynamic inversion, IEEE 5th International Conference on Cybernetics and Intelligent Systems (CIS), pp. 93- 98, 2011.
- [4]. M. Collotta, G. Pau, V.M. Salerno, G. Scata, ` A novel road monitoring approach using Wireless Sensor Networks, 6th International Conference on Complex, Intelligent, and Software Intensive Systems (CISIS), pp. 376-381, 2012.
- [5]. V. Conti, M. Collotta, G. Pau, G. Scata, S. Vitabile, ` Fuzzy techniques for access and data management in home automation environments, Journal of Mobile Multimedia, Vol. 8, Issue 3, pp. 181-203, 2012.
- [6]. M. Collotta, A. Lo Cascio, G. Pau, G. Scata, ` Smart localization platform for IEEE 802.11 industrial networks, Proceedings of SIES, pp. 69-72, 2013.
- [7]. Yi-Lin Zhou, Da-Wei Cai, Energy Saving Control System of Long Stroke Pumping Unit Based on RBF Neural Network, IHMSC Proceedings, pp. 358-361, 2012.
- [8]. M. Collotta, A. Lo Cascio, G. Pau, G. Scata, ` A fuzzy controller to improve CSMA/CA performance in IEEE 802.15.4 industrial wireless sensor networks, IEEE 18th International Conference on Emerging Technologies and Factory Automation (ETFA), pp. 1-4, 2013.
- [9]. Chen Heng, Wang Xu, Lei Tengfei, Fuzzy logic controller optimization design for tiltrotor aircraft landing conversion, 32nd Chinese Control Conference (CCC), pp. 3482-3486, 2013.

- [10]. Alaimo, A., Artale, V., Milazzo, C., Ricciardello, A., & Trefiletti, L. (2013). Mathematical modeling and control of a hexacopter. In *2013 International Conference on Unmanned Aircraft Systems (ICUAS)* (pp. 1043-1050). IEEE.
- [11]. Collotta, M., Pau, G., & Caponetto, R. (2014). A real-time system based on a neural network model to control hexacopter trajectories. In *2014 International Symposium on Power Electronics, Electrical Drives, Automation and Motion* (pp. 222-227). IEEE.
- [12]. Gupta, A. K., Jha, V., & Gupta, V. K. (2014). Design and development of remote controlled autonomous synchronic hexaroter aerial (ASHA) robot. *Procedia Technology*, 14(0), 51-58.
- [13]. Alaimo, A., Artale, V., Milazzo, C. L. R., & Ricciardello, A. (2014). PID controller applied to hexacopter flight. *Journal of Intelligent & Robotic Systems*, 73(1-4), 261-270.
- [14]. Raul M. Criado Francisco R. Rubio, "Autonomous path tracking control design for a comercial quadcopter" Published in IFAC-Papers OnLine Volume 48, Issue 9, 2015, Pages 73–78.
- [15]. E.G. Hernandez-Martinez G. Fernandez-Anaya E.D. Ferreira J.J. Flores-Godoy A. Lopez-Gonzalez, "Trajectory Tracking of a Quadcopter UAV with Optimal Translational Control" Published in IFAC-Papers OnLine Volume 48, Issue 19, 2015, Pages 226–231.
- [16]. Pietro Panizza Fabio Riccardi Marco Lovera, "Black-box and grey-box identification of the attitude dynamics for a variable-pitch quadrotor" Published in IFAC-Papers On Line Volume 48, Issue 9, 2015, Pages 61–66.
- [17]. A. S. Mostafa Moussid, "Dynamic Modeling and Control of a HexaRotor using Linear and Nonlinear Methods," *International Journal of Applied Information Systems (IJ AIS)*, Vols. Volume 9 – No.5,, August 2015.
- [18]. Andrea Alaimo, Valeria Artale, Cristina Lucia, Rosa Milazzo, Angela Ricciardello," PID Controller Applied to Hexacopter Flight" Published in *Journal of Intelligent & Robotic Systems* January 2014, Volume 73, Issue 1, pp 261-270.
- [19]. R. Bishop, *Mechatronic Systems, Sensors, and Actuators: Fundamentals and Modeling*, The Mechatronics Handbook, Second Edition ed., 2007.
- [20]. T. Magnusson, "Attitude Control of a Hexarotor," 2014